

1. [10 pts] How many real roots does each of the following polynomials possess? List all of the roots. In case there are no real roots, write NONE.

(a) $y = x^4 - 16$ Answer: number of roots = 2 ; the roots are: $x = 2, -2$

(b) $y = x^2 + 1789$ Answer: number of roots = 0 ; the roots are: do not exist

(c) $y = x^3 - 1$ Answer: number of roots = 1 ; the roots are: $x = 1$

(d) $y = (x - 9)^{99}(x + 2017)^5(x - 11)x^4$ Answer: number of roots = 4; the roots are: $x = 9, -2017, 11, 0$

(e) $y = (x^2 - 81)(x^2 + 7x + 12345)$ Answer: number of roots = 2 ; the roots are: $x = -9, 9$

2. Consider the polynomial $f(x) = (x - 8)^2(x + 3)(x - 4)^7$.

(a) [2 pts] Find the zeroes of f .

Answer: Setting $f(x) = 0$, we obtain the roots $x = -3, 4, 8$.

(b) [2 pts] Perform a sign analysis on f .

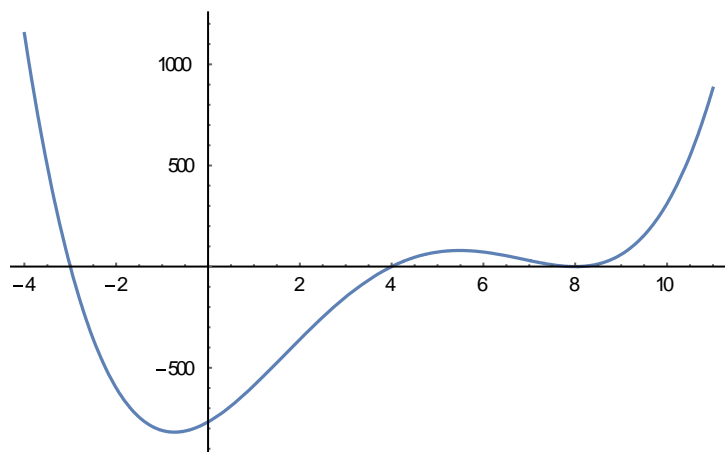
The important points for the sign test are $x = -3$ and $x = 4$.

We then discover that $y > 0$ when $x < -3$ and when $x > 4$. And y is negative on $(-3, 4)$.

(c) [2 pts] Find the limiting behavior of the graph as $x \rightarrow \infty$ and as $x \rightarrow -\infty$.

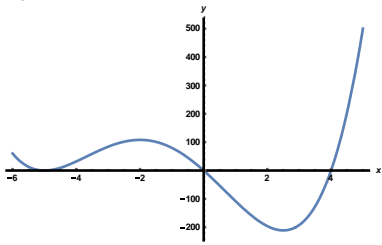
Not

(d) [5 pts] On the axes below, sketch the graph of $y = f(x)$.

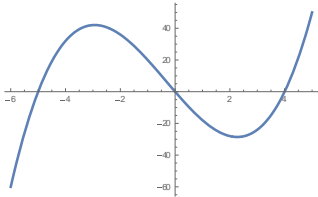


3. [8 pts] Matching

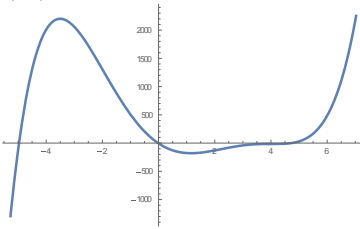
(A)



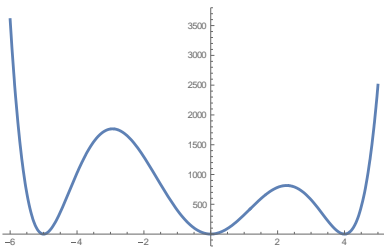
(B)



(C)



(D)



1. $y = x^2(x - 4)^3(x + 5)$

Answer: No match

2. $y = x(x - 4)^3(x + 5)$

Answer: B

3. $y = x^2(x - 4)^2(x + 5)^2$

Answer: D

4. $y = x(x - 4)(x + 5)$

Answer: B

Extra Credit Riddle: [7 pts] On the island of Oz, each of the residents is either a knight or a knave. Knights *always* tell the *truth*; Knaves *always lie*. Two residents of the island, Albertine and Beatrice, are approached by a reporter for Loyola's Phoenix. Albertine says "We are the same kind.", but Beatrice says "We are of different kinds." What, if anything, can the reporter conclude?

Solution: Albertine says "We are the same kind.", but Beatrice asserts "We are of different kinds". Clearly they are making contradictory statements and so one must be lying and one telling the truth. That is, one is a knave and one must be a knight. Since that is exactly what Beatrice said, Beatrice must be the knight, and Albertine be the knave.

*O dear Ophelia!
I am ill at these numbers:
I have not art to reckon my groans.*
- **HAMLET** (Act II, Sc. 2)