MATH 161 SOLUTIONS: QUIZ V

Problems 1 – 3 are worth 1 pt each

Hint: It may be helpful to think about $\frac{d}{dx} f(g(x))$

1. Given the graphs of the functions $f(x)$ and $g(x)$ in Figures 3.7 and 3.8, which of (a)–(d) is a graph of $f(g(x))$?

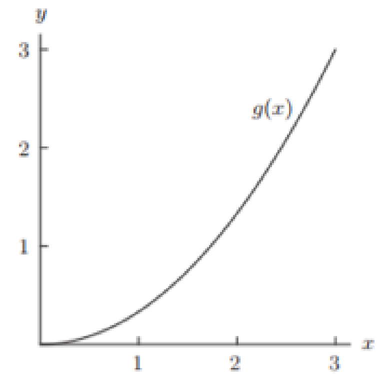


Figure 3.7

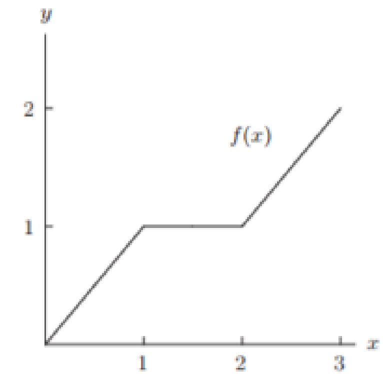
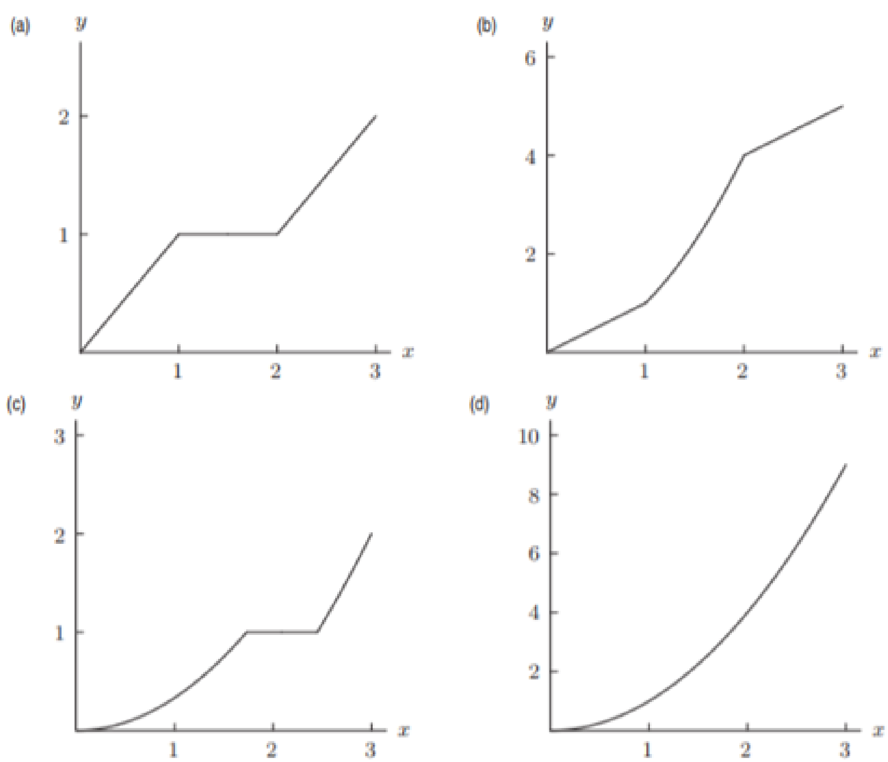


Figure 3.8



Solution: **(c)**

Because $\frac{d}{dx} f(g(x)) =$
$f'(g(x))g'(x)$, *we note that* $f(g(x))$ *has a horizontal tangent whenever* $g'(x) = 0$ *or* $f'(g(x)) = 0$.
Now $f'(g(x)) = 0$ *for* $1 < g(x) < 2$, *and this approximately corresponds to* $1.7 < x < 2.5$.

2. Given the graphs of the functions $f(x)$ and $g(x)$ in Figures 3.9 and 3.10, which of (a)–(d) is a graph of $f(g(x))$?

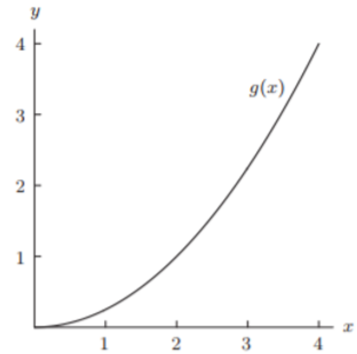


Figure 3.9

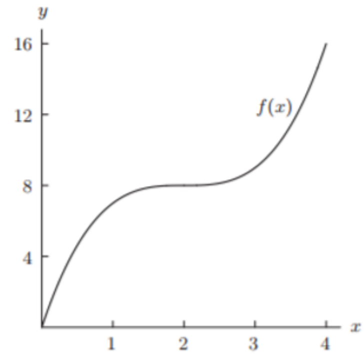
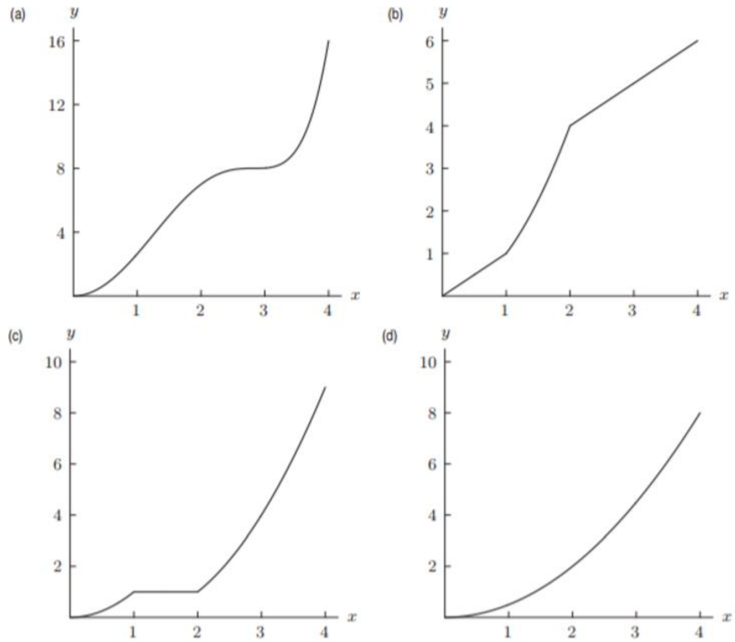


Figure 3.10



Solution: **(a)**

Because $\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$, we note that $f(g(x))$ has a horizontal tangent whenever $g'(x) = 0$ or $f'(g(x)) = 0$.

Now $f'(x) = 0$ only when $x = 2$, so the composite function has horizontal tangents only when $g'(x) = 0$ or when $g(x) = 2$.

3. Given the graphs of the function $g(x)$ and $f(x)$ in Figures 3.11 and 3.12 , which of (a)–(d) represents $f(g(x))$?

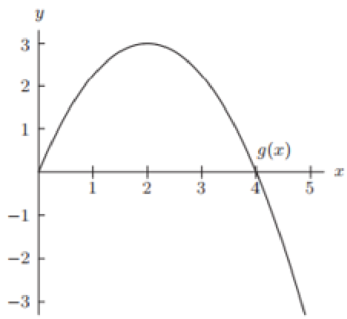


Figure 3.11

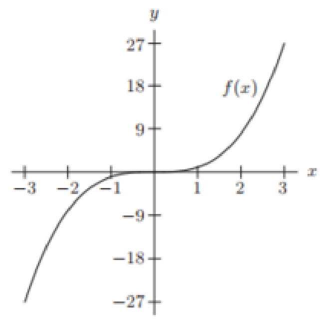
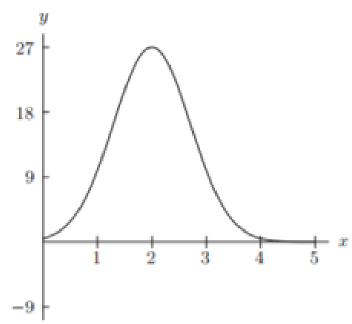
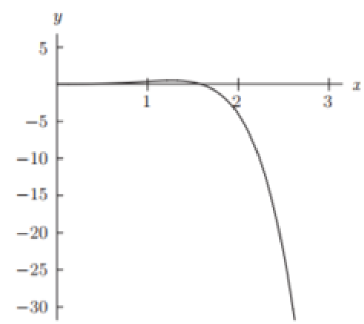


Figure 3.12

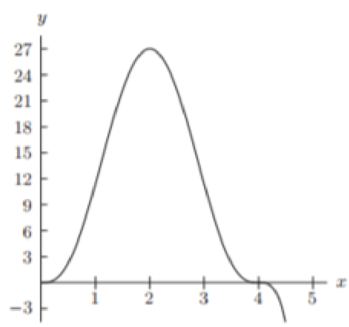
(a)



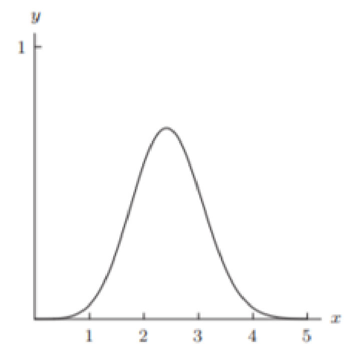
(b)



(c)



(d)



Solution: **(c)**

Because $\frac{d}{dx}f\big(g(x)\big) = f'\big(g(x)\big)g'(x)$, *we note that* $f\big(g(x)\big)$ *has a horizontal tangent whenever* $g'(x) = 0$ *or* $g(x) = 0$. This happens when x = 0, 2, and 4. Also, f(g(x)) is negative for x > 4. Alternatively f(g(4)) = f(0) = 0 identifies answer (c).

4. *[3 pts each]* Find an anti-derivative of each of the following:

(a) $1 + 3x^2 - 9x^5$

Answer: $x + x^3 - \frac{3}{2}x^6$

(b) 3 sin (5x)

Answer: $-\frac{3}{5}\cos(5x)$

(c) $1 + 3e^x + 4\cos x$

Answer: $x + 3e^x + 4\sin x$

(d) $\frac{4}{x}$

Answer: $4\ln x$

(e) $\frac{3}{1+x^2}$

Answer: $3\arctan x$

(f) $1 + \sec^2 x$

Answer: $x + \tan x$

5. *[5 pts]* Find any and all critical points of the function $f(x) = (2x-3)^3 e^x$

Solution:

$$f'(x) = (2x-3)^3\, e^x + 3(2x-3)^2 2e^x = (2x-3)^2\,(2x-3+6)e^x =$$

$$(2x+3)(2x-3)^2 e^x$$

Hence the critical points are **x = -3/2** and **x = 3/2.**

6. *[5 pt each]* Using implicit differentiation, find the equation of the *tangent line* to the curve $y^4 + xy = 4$ at the point P = (3, 1)

Solution: d/dx (LHS) = d/dx (RHS)

$$4y^3y' + xy' + y = 0$$

Substituting x = 3 and y = 2:

$$4y' + 3y' + 1 = 0$$

Hence dy/dx = -1/7 and the tangent line is:

$$y - 1 = -(1/7)(x - 3)$$

7. *[3 pts each]* Find the derivative of each of the following functions. *You need not simplify.*

(a) $y = \frac{x}{\sqrt{x-1}}$

Solution:

$$\frac{dy}{dx} = \frac{\sqrt{x-1} - x\frac{d}{dx}(x-1)^{1/2}}{x-1} = \frac{\sqrt{x-1} - x\frac{1}{2}(x-1)^{-1/2}}{x-1} =$$

$$\frac{\sqrt{x-1} - x\frac{1}{2}(x-1)^{-1/2}}{x-1} = \frac{\sqrt{x-1} - x\frac{1}{2}(x-1)^{-1/2}}{x-1} = \frac{2(x-1)-x}{(x-1)^{3/2}} = \frac{x-2}{(x-1)^{3/2}}$$

(b) $y = \arctan(1+e^{2x})$

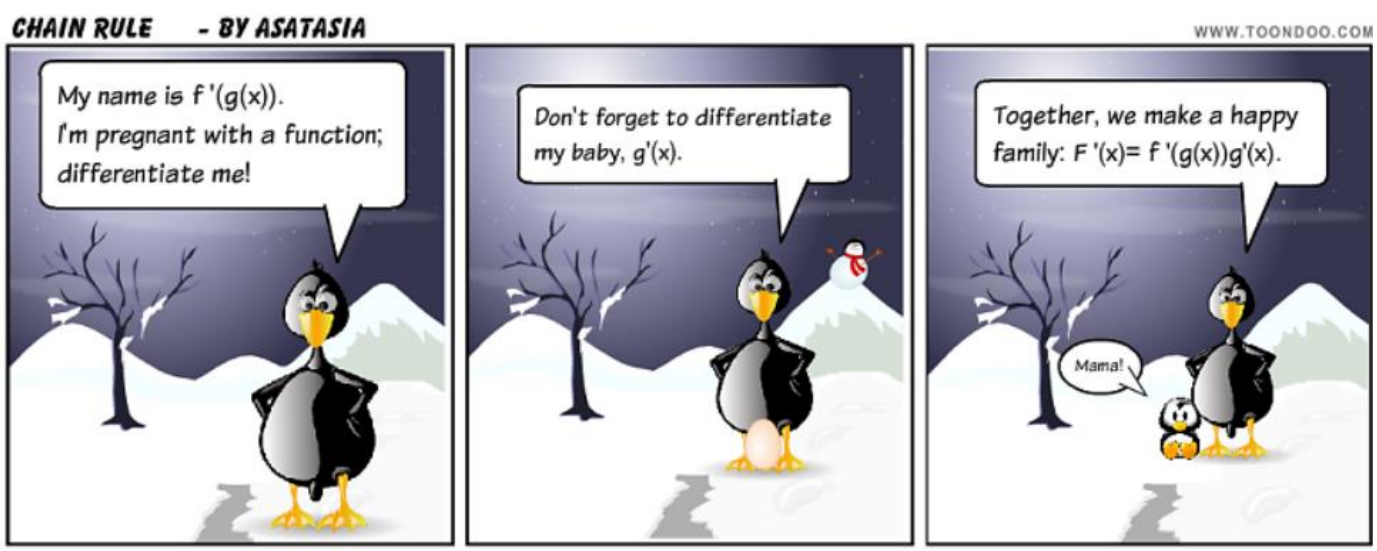
Solution: $\frac{dy}{dx} = \frac{2e^{2x}}{1+e^{4x}}$

(c) $y = \ln(\sin x)$

Solution: $\frac{dy}{dx} = \frac{\cos x}{\sin x} = \cot x$

(d) $y = \sec(1 + 3\ln x)$

Solution: $\frac{dy}{dx} = \frac{3}{x}\sec(1+3\ln x)\tan x(1+3\ln x)$



DERIVATIVE RULES

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(\sin x\right) = \cos x$$

$$\frac{d}{dx}\left(\cos x\right) = -\sin x$$

$$\frac{d}{dx}\left(a^x\right) = \ln a \cdot a^x$$

$$\frac{d}{dx}\left(\tan x\right) = \sec^2 x$$

$$\frac{d}{dx}\left(\cot x\right) = -\csc^2 x$$

$$\frac{d}{dx}\left(f(x) \cdot g(x)\right) = f(x) \cdot g'(x) + g(x) \cdot f'(x)$$

$$\frac{d}{dx}\left(\sec x\right) = \sec x \tan x$$

$$\frac{d}{dx}\left(\csc x\right) = -\csc x \cot x$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{\left(g(x)\right)^2}$$

$$\frac{d}{dx}\left(\arcsin x\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\arctan x\right) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx}\left(\operatorname{arc\,sec} x\right) = \frac{1}{x\sqrt{x^2-1}}$$