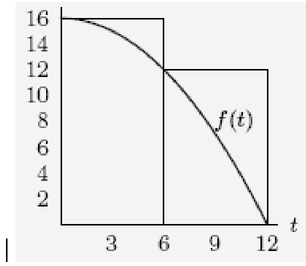
1. *[2 pts]* What does the following figure represent? *Explain briefly*.

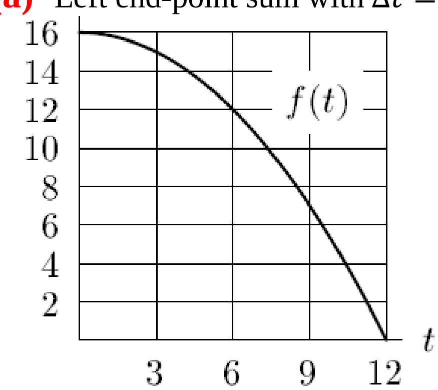


A) The right-hand Riemann sum for the function f on the interval $0 \le t \le 12$ with $\Delta t =$ 3.
B) The right-hand Riemann sum for the function f on the interval $0 \le t \le 12$ with $\Delta t =$ 6.
C) The left-hand Riemann sum for the function f on the interval $0 \le t \le 12$ with $\Delta t =$ 3.
D) The left-hand Riemann sum for the function f on the interval $0 \le t \le 12$ with $\Delta t =$ 6.

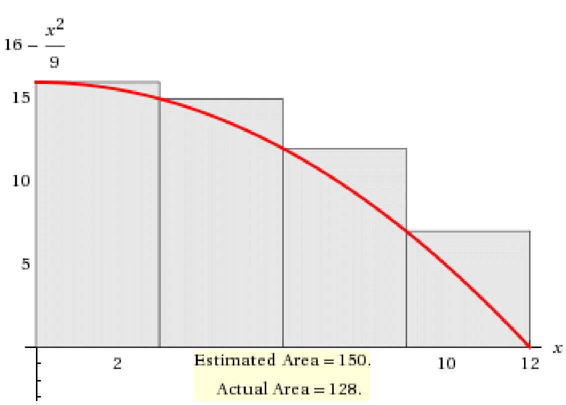
Answer: ***(D)*** *is the correct answer. The two rectangles are of width 6 and each is left-hand.*

2. *[2 pts each]* Using the given graphs, draw rectangles representing each of the following Riemann sums for the function on the interval [0, 12]. *Calculate the value of each Riemann sum.* (You may leave your answer in non-simplified form if you have no time to perform the addition.)

(a) Left end-point sum with $\Delta t = 3$.

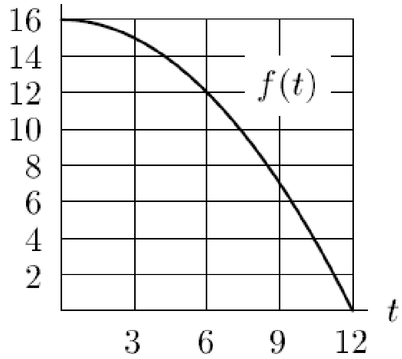


Solution:

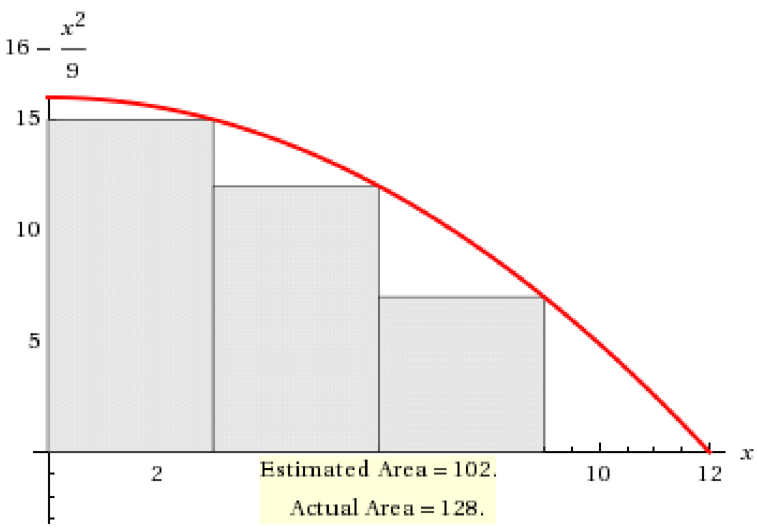


This is an overestimate.

(b) Right end-point sum with $\Delta t = 3$.

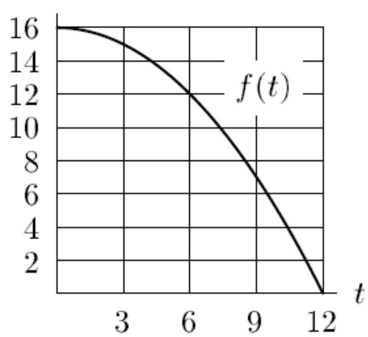


Solution:

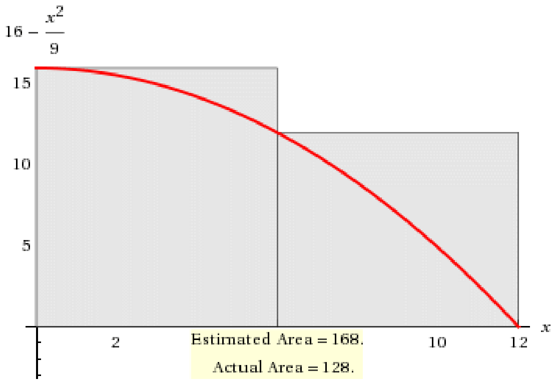


This is an underestimate.

(c) Left end-point sum with $\Delta t = 6$.

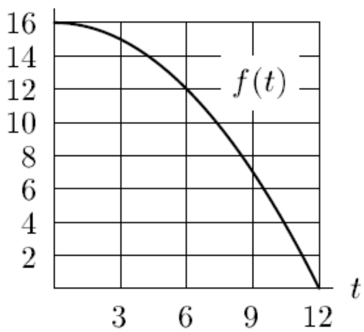


Solution:

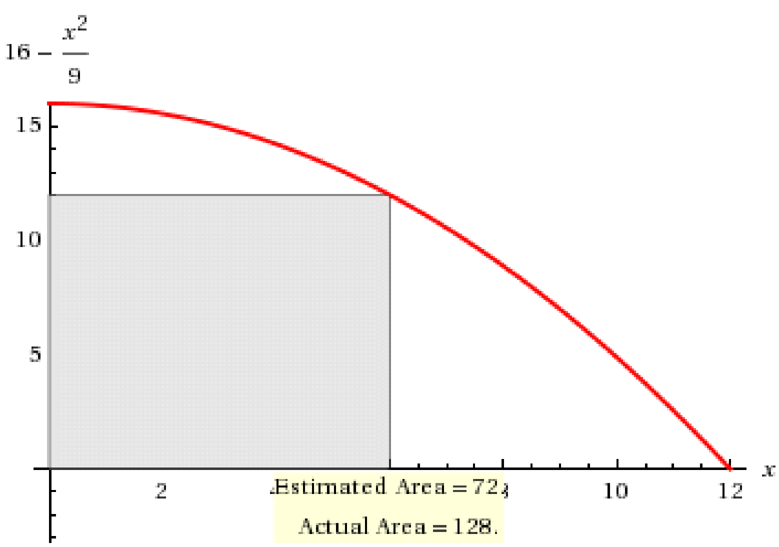


This is an overestimate.

(d) Right end-point sum with $\Delta t = 6$.



Solution:



This is an underestimate.

3. *[3 pts each]* Compute each of the following sums. *Show your work!* ***Simplify*** *your answers as much as possible.*

(a) $\sum_{j=3}^{6}(j-1)(j+1)$

Solution:

$$\sum_{j=3}^{6}(j-1)(j+1)=$$

$$(2)(4)+(3)(5)+(4)(6)+(5)(7)=8+15+24+35=82$$

(b) $\sum_{k=0}^{2017}(-1)^k$

Solution:

$$\sum_{k=0}^{7}(-2)^k =$$

$$(-1)^0+(-1)^1+(-1)^2+(-1)^3+(-1)^4+(-1)^5+\ldots+(-1)^{2016}+(-1)^{2017}=$$

$$1-1+1-1+1-1-1+\ldots+1-1=$$

$$(1-1)+(1-1)+(1-1)+\ldots+(1-1)=0$$

(c) $\sum_{n=1}^{9}\left(\frac{1}{n}-\frac{1}{n+1}\right)$

Solution:

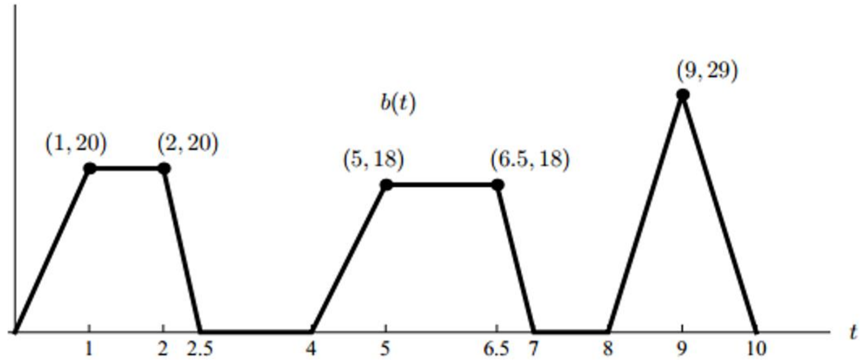
$$\sum_{n=1}^{9}\left(\frac{1}{n}-\frac{1}{n+1}\right)=$$

$$\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{6}\right)+\left(\frac{1}{6}-\frac{1}{7}\right).+\left(\frac{1}{8}-\frac{1}{9}\right)+\left(\frac{1}{9}-\frac{1}{10}\right)$$

$$=1-\frac{1}{10}=\frac{9}{10}$$

This is called a "telescoping" sum.

4. *[University of Michigan] [6 pts]* Find the (*exact* value of the) area beneath the following function over the interval [0, 10]. *Show your work!* If time permits, do the arithmetic.



Solution: Adding the areas of each of the two trapezoids and the triangle yields:

A = 35 + 40.5 + 29 = 104.5 square units

Alternatively, one can compute the area of five triangles and two rectangles.

5. *[5 pts]* Evaluate $\int (1 + 2x - \cos x + e^{4x})\, dx$

Solution:

$$\int (1+2x-\cos x+e^{4x})\,dx = \int 1\,dx + \int 2x\,dx - \int \cos x\,dx + \int e^{4x}\,dx =$$
$$x + x^2 - \sin x + (1/4)e^{4x} + C$$

6. *[5 pts]* Albertine *claims that* the following anti-differentiation formula is correct:

$$\int x^3 e^x\, dx = \left(x^3 - 3x^2 + 6x - 6\right) e^x + C$$

Determine whether Albertine is correct or mistaken. *You must show your work to earn credit.*

Solution: Differentiating the right-hand side (using the product rule)

$$\frac{d}{dx}\left(\left(x^3 - 3x^2 + 6x - 6\right) e^x + C\right) =$$
$$\left(x^3 - 3x^2 + 6x - 6\right)e^x + \left(3x^2 - 6x + 6\right)e^x + 0 =$$
$$\left(\left(x^3 - 3x^2 + 6x - 6\right) + \left(3x^2 - 6x + 6\right)\right)e^x =$$
$$x^3 e^x$$

Hence Albertine is correct!

7. *[5 pts]* Solve the following initial value problem:

$$\frac{dy}{dx} = (1+3x)^{48} + x + 5; \quad y(0) = \frac{1}{98}$$

Solution: Using judicious guessing,

$$y = \int ((1+3x)^{48} + x + 5)\, dx = \frac{(1+3x)^{49}}{49(3)} + \frac{x^2}{2} + 5x + C.$$

Using the initial condition, $\frac{1}{98} = \frac{(1+3(0))^{49}}{49(3)} + \frac{0}{2} + 5(0) + C = \frac{1}{49(3)} + C$

Hence $C = \frac{1}{98} - \frac{1}{49(3)} = \frac{1}{2(49)} - \frac{1}{49(3)} = \frac{1}{49}\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{49(6)} = \frac{1}{294}.$

Finally, the solution to the initial value problem is

$$\boldsymbol{y = \frac{(1+3x)^{49}}{147} + \frac{x^2}{2} + 5x + \frac{1}{294}}.$$

Common integration is only the memory of differentiation.

- Augustus de Morgan

Augustus De Morgan (1806-1871)

DERIVATIVE RULES

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(\sin x\right) = \cos x$$

$$\frac{d}{dx}\left(\cos x\right) = -\sin x$$

$$\frac{d}{dx}\left(a^x\right) = \ln a \cdot a^x$$

$$\frac{d}{dx}\left(\tan x\right) = \sec^2 x$$

$$\frac{d}{dx}\left(\cot x\right) = -\csc^2 x$$

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) = f(x)\cdot g'(x) + g(x)\cdot f'(x)$$

$$\frac{d}{dx}\left(\sec x\right) = \sec x \tan x$$

$$\frac{d}{dx}\left(\csc x\right) = -\csc x \cot x$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{\left(g(x)\right)^2}$$

$$\frac{d}{dx}\left(\arcsin x\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\arctan x\right) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\cdot g'(x)$$

$$\frac{d}{dx}\left(\operatorname{arc\,sec} x\right) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(\ln x\right) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\sinh x\right) = \cosh x$$

$$\frac{d}{dx}\left(\cosh x\right) = \sinh x$$