# MATH 161 Solutions: Quiz ix

# 10 November 2017

1. Evaluate each of the following Riemann integrals using only basic geometry. *Explain your reasoning! Give an exact answer. [4 pts each]*



*Solution:* 



This Riemann integral represents the area of a rectangle of height 1 and width 3 plus the area of a triangle with base 3 and height 24.

Thus A = 1(3) + ½ (3)(24) = **39**



*Solution:*



Now the integrand of the second integral is that of a semi-circle of radius 4.

Hence A = 3 ½  42 = **24**





*Solution:* Here the integrand represents a region determined by two triangles.

So A = ½ (10)(10) + ½ (2)(2) = **52**



2. Let f(x) = 4x3 – 8x2 + 7x – 2 be defined on the interval [2, 5].

1. *[2 pts]* Explain why y = f(x) satisfies the hypotheses of the Mean Value Theorem.

*Answer: Since f is a polynomial, it is continuous on [2, 5] and differentiable on (2, 5).*

1. *[4 pts]* Find any and all numbers, *c*, that satisfy the conclusion of the Mean Value Theorem. Express your answer(s) to the nearest hundredth.

*Solution: Computing: f(5) – f(2) = = 107*

*Now: *

*We must solve the equation: .*

*Now = 107, or equivalently: .*

*Dividing both sides by 4: .*

*Factoring: (3c – 10)(c + 2) = 0.*

*Thus c = 10/3, c = -2. We reject the root c = -2 since it doesn’t lie within the domain of f.*

***So c = 10/3*** $≈3.33$

*See the graph below.*



3. The graph in the figure below is the graph of $ \frac{dh}{dt}$, where *h* is the altitude in thousands of feet above sea level and *t* is in hours, for Albertine's recent climb to the top of Bear Peak in Colorado. Use the graph to answer the following questions. Show your work! *[3 pts each]*



1. How long did it take Albertine to reach the peak of the mountain?

*Solution: h(t) is increasing on the interval [0, 4]. Thus Albertine took* ***4 hours*** *to reach the summit of the mountain.*

1. What was the total change in altitude between t = 0 and t = 4?

*Solution: h(4) – h(0) = area beneath the curve dh/dt that lies above [0, 4]. This may be thought of as the area of a rectangle plus the area of a triangle.*

*Hence, change in altitude is (0.8) (2) + ½ (2)(0.8) =* ***2.4 thousand feet****.*

1. If Albertine began her climb at 6000 feet above sea level, how high is the peak above sea level?

*Solution: The peak is 6000 + 2400 =* ***8400 feet above sea level.***

1. After 6 hours, Albertine stopped at a lookout point to have a snack. What was the altitude of the lookout point?

*Solution: Albertine’s descent from t = 4 to t = 6 is ½(2)(0.8) = 0.8 thousand feet.*

*Hence, at t = 6, Albertine’s altitude is 6000 + 2400 – 800 =* ***7600 feet****.*

4. Below is the graph of the *velocity*, in feet per second, 0 ≤ t ≤ 13, of a frightened skunk as it tries to run away from a German Shepard. From t = 0 to t = 2, we have a quarter of a circle; from t = 2 to t = 8, we have a semicircle; from t = 8 until t = 13, we have a straight line segment.

*[Note that this is the graph of velocity, not distance.]*



1. *[5 pts]* What is the *total distance* traveled by the skunk between t = 0 and t = 13 seconds? Give an answer correct to the nearest hundredth.

*Solution: The distance traveled by the skunk is the area under the velocity curve between t = 0 and t = 13 seconds. This sum consists of the area of a quarter circle of radius 2 + the area of a half-circle of radius 3 + the area of a triangle with base length 5 and height 10.*

*Thus the total distance = (1/4)  (2)2 + (½) (3)2 + ½ (5)(10) =* ***5.****feet* ***42.28 feet.***

1. *[3 pts]* What is the *average velocity* traveled by the skunk between t = 0 and t = 13 seconds? Give an answer correct to the nearest hundredth.

*Solution: The average velocity from t = 0 to t = 13 seconds is the change in position (which, in this exercise, equals the total distance traveled from t = 0 to t = 13) divided by 13, viz:*

***(5.5****feet/sec* ***3.25 feet/sec***

1. *[3 pts]* What is the *average velocity* traveled by the skunk between t = 2 and t = 10 seconds? Give an answer correct to the nearest hundredth.

*Solution: The average velocity from t = 2 to t = 10 seconds is the change in position (which, in this exercise, equals the total distance traveled from t = 2 to t = 10) divided by 8.*

*Now the total distance is the sum of the area of a half-circle of radius 3 and the area of a triangle with base length 2 and height 4, viz:*

*(4.5 feet/sec* ***2.27 feet/sec***

 

Extra Credit:

Consider the graph of y = f(x) defined on the interval [0, 6] as represented below.

 



1. What is the *minimum* value of G(x) on the interval [0, 6] and where is that value achieved? *Explain your reasoning.*

*Solution: Note that G is a function of x alone!*

*Using the area interpretation of the definite integral, one can see that the integral is* ***minimized when x = 1.5****. Now*



1. What is the *maximum* value of G(x) on the interval [0, 6] and where is that value achieved? *Explain your reasoning.*

*Solution: Note that G is a function of x alone!*

*Using the area interpretation of the Riemann integral, one can see that the integral is* ***maximized when x = 4.5****. Now*

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*Thus the* ***maximum*** *value* ***achieved*** *by G(x) on the interval [0, 6] is* ***1.25****.*

#  “Mario, what do you get when you cross an insomniac, an unwilling agnostic and a dyslexic?"

# "I give."

# "You get someone who stays up all night torturing himself mentally over the question of whether or not there's a dog.”

― [David Foster Wallace](https://www.goodreads.com/author/show/4339.David_Foster_Wallace), [**Infinite Jest**](https://www.goodreads.com/work/quotes/3271542)