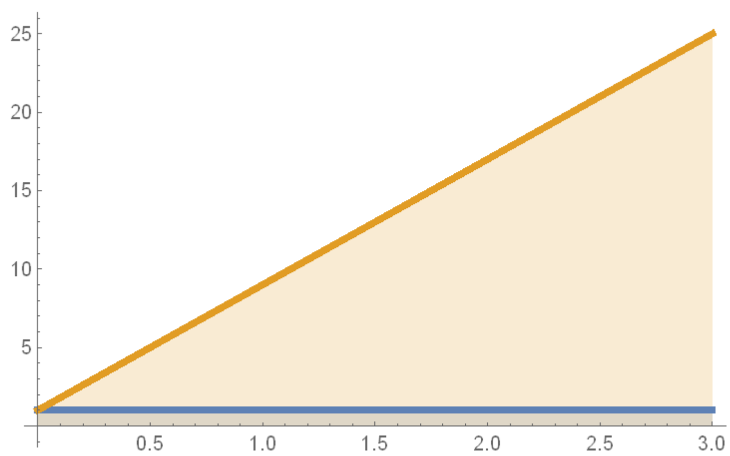
MATH 161 SOLUTIONS: QUIZ IX

10 NOVEMBER 2017

1. Evaluate each of the following Riemann integrals using only basic geometry. *Explain your reasoning! Give an exact answer. [4 pts each]*

$$(a) \quad \int_0^3 (1+8x)\ dx$$

Solution: $\displaystyle\int_0^3 (1+8x)\ dx$



This Riemann integral represents the area of a rectangle of height 1 and width 3 plus the area of a triangle with base 3 and height 24.

Thus A = 1(3) + ½ (3)(24) = **39**

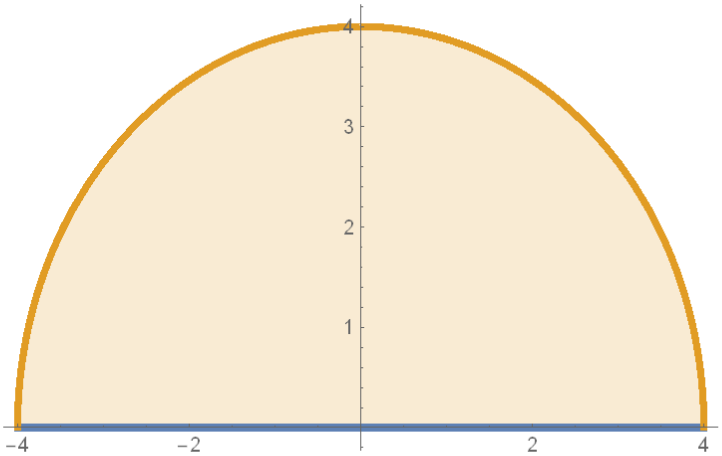
$$(b) \quad \int_{-4}^{4} 3\sqrt{16-t^2}\ dt$$

Solution:

$$\textit{Note that } \int_{-4}^{4} 3\sqrt{16-t^2}\ dt = 3\int_{-4}^{4} \sqrt{16-t^2}\ dt$$

Now the integrand of the second integral is that of a semi-circle of radius 4.

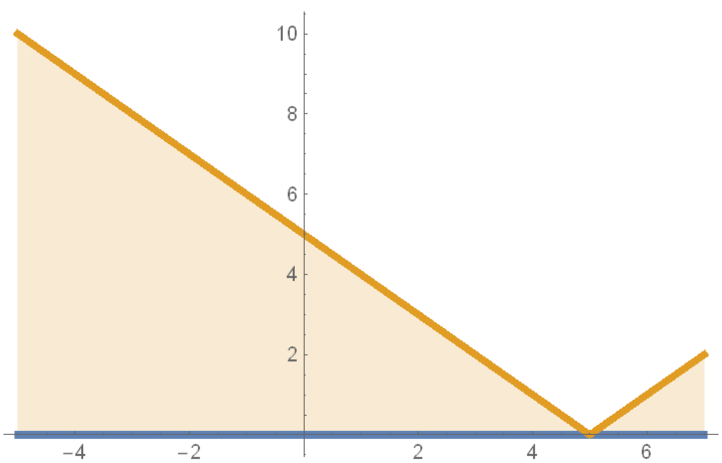
Hence A = 3 ½ π 4^2 = **24π**



(c) $$\int_{-5}^{7} |z-5|\,dz$$

Solution: Here the integrand represents a region determined by two triangles.

So A = ½ (10)(10) + ½ (2)(2) = **52**



2. Let f(x) = $4x^3 - 8x^2 + 7x - 2$ be defined on the interval [2, 5].

(a) *[2 pts]* Explain why y = f(x) satisfies the hypotheses of the Mean Value Theorem.

Answer: Since f is a polynomial, it is continuous on [2, 5] and differentiable on (2, 5).

(b) *[4 pts]* Find any and all numbers, *c*, that satisfy the conclusion of the Mean Value Theorem. Express your answer(s) to the nearest hundredth.

Solution: Computing: f(5) – f(2) = = 107

Now: $f'(x) = 12x^3 - 16x^2 + 7$

We must solve the equation: $f'(c) = \dfrac{f(5) - f(2)}{5-2} = \dfrac{333-12}{3} = 107$.

Now $f'(c) = 12c^3 - 16c^2 + 7$ *= 107, or equivalently:* $12c^3 - 16c^2 - 100 = 0$.

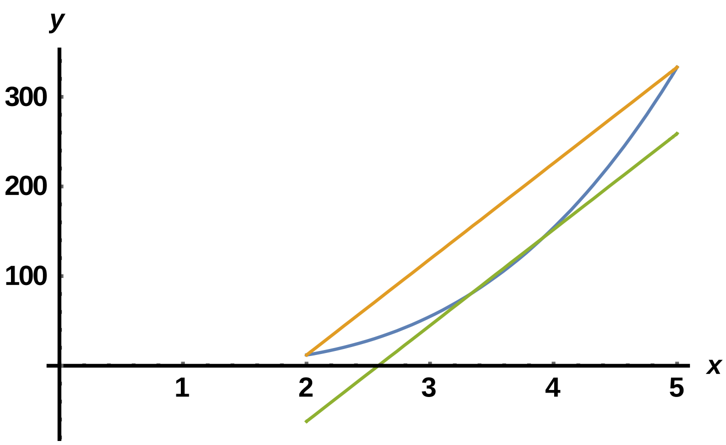
Dividing both sides by 4: $3c^3 - 4c^2 - 20 = 0$.

Factoring: $(3c - 10)(c + 2) = 0$.

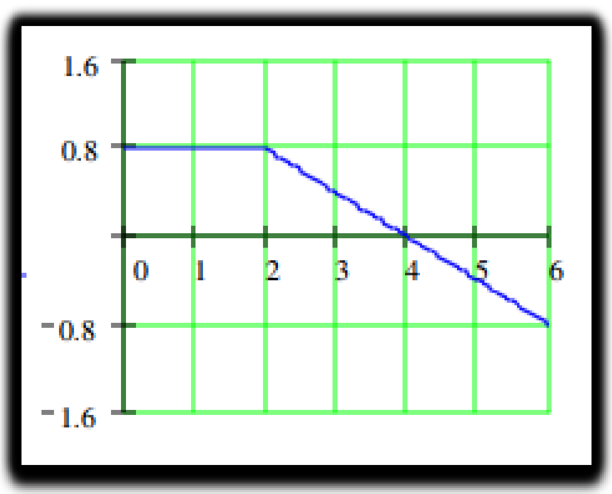
Thus $c = 10/3$, $c = -2$. *We reject the root* $c = -2$ *since it doesn't lie within the domain of f.*

So $c = 10/3 \approx 3.33$

See the graph below.



3. The graph in the figure below is the graph of $\frac{dh}{dt}$, where h is the altitude in thousands of feet above sea level and t is in hours, for Albertine's recent climb to the top of Bear Peak in Colorado. Use the graph to answer the following questions. Show your work! *[3 pts each]*



(a) How long did it take Albertine to reach the peak of the mountain?

*Solution: h(t) is increasing on the interval [0, 4]. Thus Albertine took **4 hours** to reach the summit of the mountain.*

(b) What was the total change in altitude between t = 0 and t = 4?

Solution: h(4) – h(0) = area beneath the curve dh/dt that lies above [0, 4]. This may be thought of as the area of a rectangle plus the area of a triangle.
*Hence, change in altitude is (0.8) (2) + ½ (2)(0.8) = **2.4 thousand feet**.*

(c) If Albertine began her climb at 6000 feet above sea level, how high is the peak above sea level?

*Solution: The peak is 6000 + 2400 = **8400 feet above sea level.***

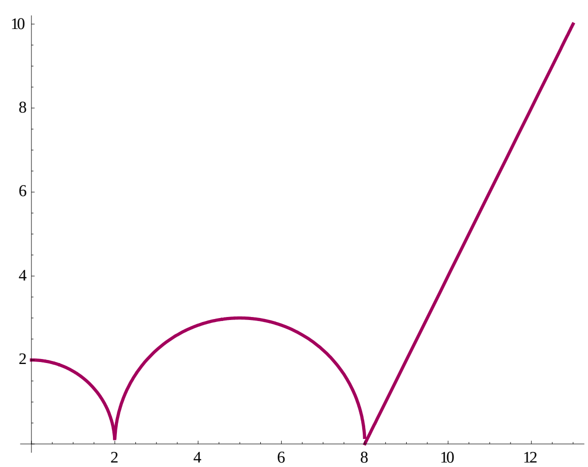
(d) After 6 hours, Albertine stopped at a lookout point to have a snack. What was the altitude of the lookout point?

Solution: Albertine's descent from t = 4 to t = 6 is ½(2)(0.8) = 0.8 thousand feet.

*Hence, at t = 6, Albertine's altitude is 6000 + 2400 – 800 = **7600 feet**.*

4. Below is the graph of the *velocity*, in feet per second, $0 \le t \le 13$, of a frightened skunk as it tries to run away from a German Shepard. From t = 0 to t = 2, we have a quarter of a circle; from t = 2 to t = 8, we have a semicircle; from t = 8 until t = 13, we have a straight line segment.

[Note that this is the graph of velocity, not distance.]



(a) *[5 pts]* What is the *total distance* traveled by the skunk between t = 0 and t = 13 seconds? Give an answer correct to the nearest hundredth.

Solution: The distance traveled by the skunk is the area under the velocity curve between t = 0 and t = 13 seconds. This sum consists of the area of a quarter circle of radius 2 + the area of a half-circle of radius 3 + the area of a triangle with base length 5 and height 10.

Thus the total distance = (1/4) $\pi(2)^2$ + (½)$\pi(3)^2$ + ½ (5)(10) = ***$5.5\pi + 25$*** *feet ≈* ***42.28 feet.***

(b) *[3 pts]* What is the *average velocity* traveled by the skunk between t = 0 and t = 13 seconds? Give an answer correct to the nearest hundredth.

Solution: The average velocity from t = 0 to t = 13 seconds is the change in position (which, in this exercise, equals the total distance traveled from t = 0 to t = 13) divided by 13, viz:

$$(5.5\pi + 25)/13 \text{ feet/sec} \approx \mathbf{3.25 \text{ feet/sec}}$$

(c) *[3 pts]* What is the *average velocity* traveled by the skunk between t = 2 and t = 10 seconds? Give an answer correct to the nearest hundredth.

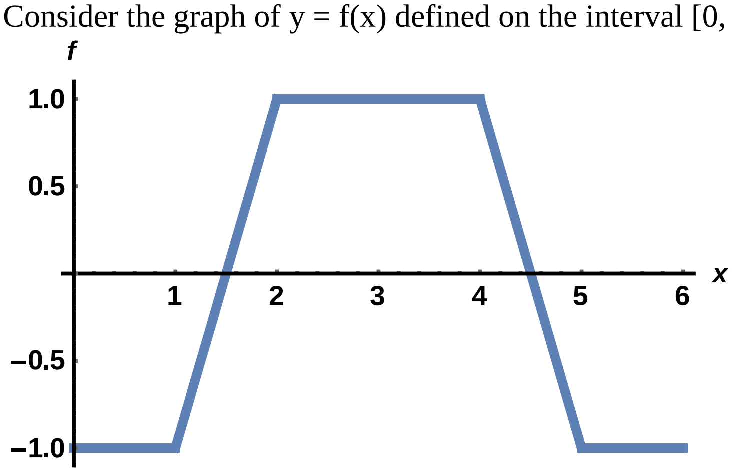
Solution: The average velocity from t = 2 to t = 10 seconds is the change in position (which, in this exercise, equals the total distance traveled from t = 2 to t = 10) divided by 8.

Now the total distance is the sum of the area of a half-circle of radius 3 and the area of a triangle with base length 2 and height 4, viz:

$$(4.5\pi + 4)/8 \text{ feet/sec} \approx \mathbf{2.27 \text{ feet/sec}}$$

EXTRA CREDIT:

Consider the graph of y = f(x) defined on the interval [0, 6] as represented below.



$$\text{Let } G(x) = \int_0^x f(t)\,dt \text{ be defined for } 0 \le x \le 6.$$

(a) What is the *minimum* value of G(x) on the interval [0, 6] and where is that value achieved? *Explain your reasoning.*

Solution: Note that G is a function of x alone!

Using the area interpretation of the definite integral, one can see that the integral is ***minimized when x = 1.5****. Now*

$$G(1.5) = \int_0^{1.5} f(t)\,dt = \int_0^{1} f(t)\,dt + \int_1^{1.5} f(t)\,dt =$$
$$-1 - 0.25 = -1.25$$

(b) What is the *maximum* value of G(x) on the interval [0, 6] and where is that value achieved? *Explain your reasoning.*

Solution: Note that G is a function of x alone!

Using the area interpretation of the Riemann integral, one can see that the integral is ***maximized when x = 4.5****. Now*

$$G(4.5) = \int_0^{4.5} f(t)\,dt = \int_0^{1} f(t)\,dt + \int_1^{2} f(t)\,dt + \int_2^{4} f(t)\,dt + \int_4^{4.5} f(t)\,dt =$$
$$-1 + 0 + 2 + (1/2)(1/2)(1) = 1.25$$

Thus the ***maximum*** *value* ***achieved*** *by G(x) on the interval [0, 6] is* ***1.25****.*

"Mario, what do you get when you cross an insomniac, an unwilling agnostic and a dyslexic?"

"I give."

"You get someone who stays up all night torturing himself mentally over the question of whether or not there's a dog."

— David Foster Wallace, **Infinite Jest**