# MATH 161 Solutions: TEST II 20 October 2016

*Instructions: Answer any 9 of the following 11 questions. You may answer more than 9 to earn extra credit. Each problem is worth 13 pts.*



*The calculus is one of the greatest edifices constructed by mankind.*

- Cambridge Conference on School Mathematics

1. Differentiate each of the following functions. You *need not* simplify. Show your work.
2.

*Solution: Using the chain rule,*

1. y =

*Solution: Using the quotient rule,*

1.

*Solution: Using the product rule,*

1.

*Solution: Using the chain rule,*

*2.*  Using the method of judicious guessing, find an *anti-derivative* for each of the following expressions. *You need not simplify your answers.* Show your work.

1. cos x + cosh 3x + sec2 x + 3 sec x tan x

**Answer:**

1.

**Answer:**

1.

**Answer:**

(d) 

**Answer:**

(e)

**Answer:**

3. Consider the curve y = x3e-4x. Perform all three “stages” in the curve-sketching process. *Show your work!* On the graph, identify all local and global extrema and points of inflection. Calculate *explicitly* the x-coordinates of each such point (i.e., local extrema and inflection pts).

***Hint:*** Albertine wants you to succeed! She has computed the first and second derivatives for you!

*Solution:*

*dy/dx = 0 when x = 0, x = 4/3*

*Thus the critical points occur at x = 0 and x = 4/3.*

*Using the quadratic formula, the latter occurs when*



**Answers:**

*zero(s): x = 0*

*asymptote(s): y = 0*

*critical point(s): x = 0, 3/4*

*behavior as x → ∞: y → 0*

*behavior as x → - ∞: y → - ∞*

*local max at x = 3/4*

*local min: none*

*global max at x = 3/4*

*global min: none*

*inflection point(s) x = 0, 0.32, 1.18*

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4. *[Stewart]* The altitude of a triangle is increasing at a rate of 2 cm/min while the area of the triangle is increasing at a rate of 3 cm2/min. At what rate is the base of the triangle changing when the altitude is 10 cm and the area is 100 cm2? *(As usual, introduce your variables, state what is given, and state your objective.)*

*Solution:*

***Variables:*** *Let b denote the base length of the triangle in cm. Let h denote the altitude length in cm. Let A denote the area of the triangle.*

***Given:*** *dA/dt = 3 and dh/dt = 2*

***Goal:*** *Find db/dt when h = 10 and A = 100.*

*Toward this end, we use the product rule:*

*Since A = ½ b(t) h(t),*

**

*and so, when h = 10 and A = 100, we have b = 20 and so:*

**

*Solving for db/dt yields: db/dt = (6 – 40)/10 =* ***-17/5 = -3.4 cm/min****.*

*5. (a) Find* dy/dx if exy = x + y. (*Hint:* Use logarithmic differentiation.)

*Solution:*

 x

x

x

(b) Find dy/dx if xy = yx.

*Solution:*

***6.***  Let g(x) = x3 (x – 2)2 be defined on [-1, 2.2].

(a) *[2 pts]* Why must g possess a global max and a global min?

*Answer: Since g(x) is continuous on [-1, 2.2], a closed and bounded interval, the* ***Extreme Value Theorem*** *guarantees the existence of a global max and a global min on [-1, 2.2].*

(b) *[3 pts]* Find all the *critical points* of *g*.

*Solution: Differentiating:*

*g' (x) = x3 2(x – 2) + 3x2 (x – 2)2 = x2(x – 2) (2x + 3(x – 2)) =*

*x2(x – 2) (5x – 6)*

*Setting g' (x) = 0, we obtain the critical points x =* ***0, 6/5,*** *and* ***2****.*

*(c) [4 pts]* *Classify* each of the *critical points* and *endpoints* as local max, local min, or neither.

*Solution: Endpoint x = -1 is a local and global minimum*

*x = 0 is neither*

*x = 6/5 is a local and global maximum since g(1.2) > g(2.2)*

*x = 2 is a local minimum*

*Endpoint x = 2.2 is a local maximum*

*Endpoint x = -1 is a local and global minimum*

1. *[4 pts]* Sketch a graph of y = g(x). Label all local and global extrema. Show regions of increase and decrease. Show regions where the function is concave up and concave down.

*Solution:*



*Arrows point to global max and global min. (These are also local extrema.) The other local extrema are indicated by the symbol*

7. The curve given by the equation x2 + y2 = (2x2 + 2y2 – x)2 is called a *cardioid*. (Its graph is represented below.) Using implicit differentiation, find an equation of the *tangent line* to this curve at the point P = (0, ½).



*Solution:*

*Differentiating implicitly, we obtain:*

*2x + 2y(dy/dx) = 2(2x2 + 2y2 – x)(4x + 4y(dy/dx) – 1)*

*Next, letting x = 0 and y = ½ we find:*

*2(0) + 1(dy/dx) = 2(0 + ½ – 0)(0 + 2(dy/dx) – 1)*

*Hence: dy/dx = 2(dy/dx) – 1 and so dy/dx = 1.*

*So the equation of the tangent line to the curve at P is:*

*y – ½ = 1(x – 0) or, equivalently,* ***y = x + ½***

8. *[Stewart]* A water trough is 10 ft. long and a cross-section has the shape of an isosceles trapezoid that is 30 cm wide at the bottom, 80 cm wide at the top, and has height 50 cm. If the trough is being filled with purified water at the rate of 0.2 m3/min, how fast is the water level rising when the water is 30 cm deep?

*(As usual, introduce your variables, state what is given, and state your objective.)*

|  |  |
| --- | --- |
| Résultats de recherche d'images pour « isosceles trapezoid » | Résultats de recherche d'images pour « isosceles trapezoid trough » |

*Solution:*

**9.** Sketch the graph of the function  defined on the domain x > 0. Find the x-coordinates of *all local and global extrema* (if any) as well as *inflection points* (if any). On your graph label all local/global extrema and inflection points. ***Show your work!***

*Solution:*



10. Let *f* (*x*) be a differentiable function defined for all real *x* with ***derivative***

 *ex−*1 *x*4(*x* + 4) (*x −* 3)2*.*

* 1. Find the *x*-coordinates of all *critical points* of *f* (*x*).

*Solution: The critical points occur at x = 0, x = -4, x = 3*

* 1. Find the *x*-coordinates of all local extrema of *f* (*x*). If there are none of a particular type, write **none**.

*Justify your answers, and be sure to show enough evidence to demonstrate that you have*

*found all local extrema.*

*Solution: Performing a sign analysis on*

*So*

*Thus f is increasing when x>-4 and decreasing when x <-4.*

*We now conclude that f has a local minimum at x = -4. There are no other local extrema.*

* 1. Is there a global max? If so, where?

*Solution: From part (b) it is clear that there is no global maximum as there is no local maximum.*

* 1. Is there a global min? If so, where?

*Solution: From part (b) it is clear that f must have a global minimum at x = -4.*

11*. [University of Michigan]* For positive *A* and *B*, the force between two atoms is a function of the distance, *r*, between them:

  *r >* 0*.*

* 1. Find the zeroes of *f* (in terms of *A* and *B*).

*Solution: Finding a common denominator for f, we have*

*Setting f(r) = 0, we have is the only zero of f.*

* 1. Find the coordinates of the critical points and inflection points of *f* in terms of *A* and *B*.

*Solution: To find the critical points we set*



* 1. If *f* has a local minimum at (1*,* −2) find the values of *A* and *B*. Using your values for *A* and *B*, justify that (1*,* −2) is a local minimum.



