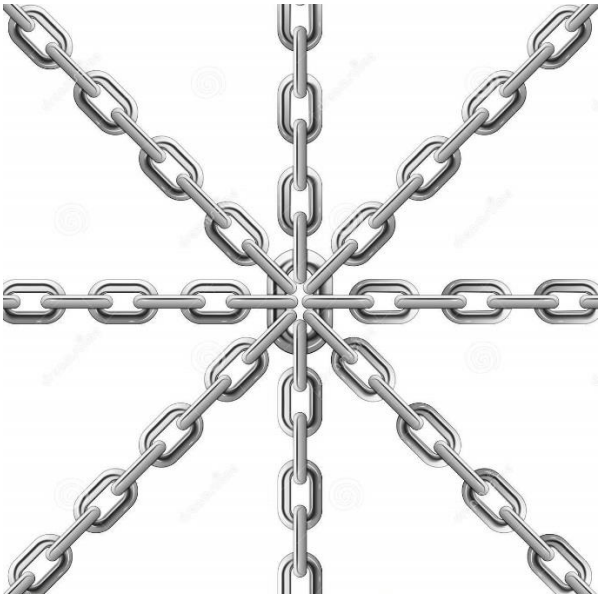
WORKSHEET X

Chain Rule, Implicit Differentiation, Logarithmic Differentiation,

Inverse Trig functions



I Compute dy/dx using the Chain Rule:

1. $y = (1+\sin x)^8$
2. $y = \sqrt{5+x^3+2x^5}$
3. $y = e^{1+\cos x}$
4. $y = \dfrac{e^x - e^{-x}}{e^x + e^{-x}} = \tanh x$
5. $y = \sin(13\cos x)$
6. $y = e^{4x}\tan 5x$
7. $y = \sin^4 x + \sqrt{3x+11}$
8. $y = (x+1)^5(3x-13)^7$

9. $y = \dfrac{\sec 3x}{\sqrt{2x+1}}$

10. $y = \sec(x + \ln x)$

II For each of the following curves, find all *critical points* (i.e., points for which dy/dx = 0).

1. $y = (x+1)^5(2x-1)^8$
2. $y = e^{-3x}(x+4)^9$
3. $y = \dfrac{(3x-5)^5}{(2x+1)^3}$
4. $y = x + \sin x$
5. $y = 13x + 3\sin 4x$

III 1. Given $y = \tan^2(\pi u/8)$ and $u = 1 + 2x^2 - 4x^3 + 3$, find dy/dx when x = 1.

2. Sketch the curve $y = (2x - 1)^4(3x + 1)^5$ and locate all zeroes, perform a sign analysis, study limiting behavior and locate all critical points.
3. Sketch the curve $y = e^x(x - 1)^4$ and locate all zeroes, perform a sign analysis, study limiting behavior and locate all critical points.
4. Show that the derivative of ln x is 1/x. (*Hint:* Let y = ln x; then $x = e^y$.)
5. Find dy/dx if y = ln(sec x + tan x) and simplify your answer.
6. Find dx/dt if x(t) = ln(ln(t)).

IV Using implicit differentiation, find dy/dx:

1. $y + x = xy + 7$
2. $y^2 = x^2 + \sin xy$
3. $y \sin \dfrac{1}{y} = 1 - xy$

V 1. Prove the power rule for *rational* exponents, *viz.*

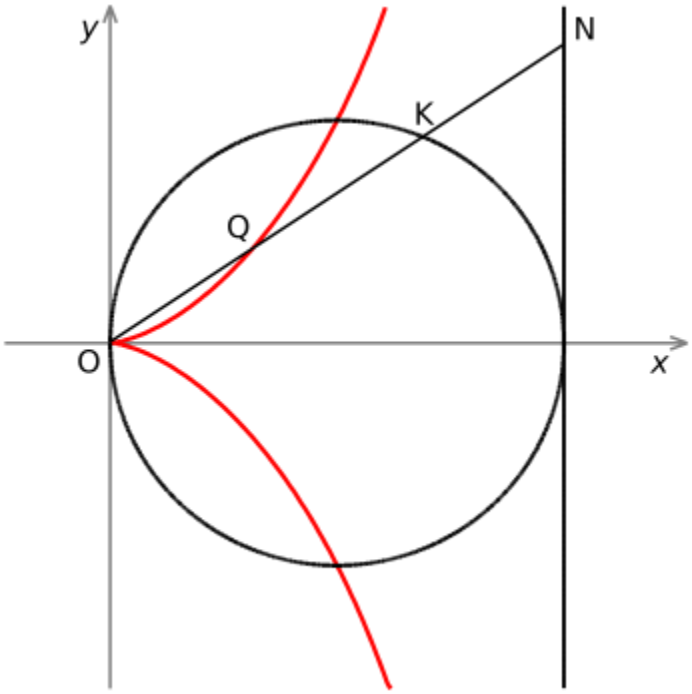
$(d/dx)\, x^p = px^{p-1}$ if *p* is rational.

2. Find d^2y/dx^2 if $y^2 + xy = 1$.

3. Consider the curve defined implicitly by: $x^2 + xy - y^2 = 1$. Verify that the point P = (2, 3) lies on this curve. Find the equations of the *tangent* and *normal* lines to this curve at the point *P*.

4. Find equations for the *tangent* and *normal* lines to the *cissoid of Diocles* (from 200 B.C.):

$y^2(2 - x) = x^3$ at Q = (1, 1).



VI Using implicit differentiation, find dy/dx for each of the following inverse trig functions.

y = arcsin x, y = arctan x, and y = arcsec x.

VII Find dy/dx for each of the following:

1. $y = \arcsin(2x + 5)$
2. $y = \arctan\left(\frac{1}{x}\right)$

3. $y = \ln(arc\sec x)$

4. $y = \left(\arcsin(x^2)\right)^5$

VIII Using logarithmic differentiation, find dy/dx for each of the following:

1. $y = x(x+1)^5(3x-4)^{11}$

2. $y = \dfrac{5x+7}{\sqrt{3x+5}}$

3. $y = \sqrt{\dfrac{x(3x+1)(2x+5)}{(x-4)(7x-1)}}$

It is often better to be in chains than to be free.

- Franz Kafka, **The Trial**