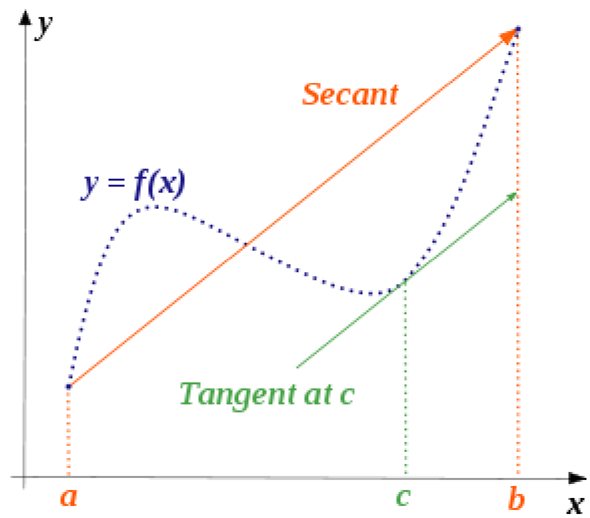
WORKSHEET XVI

MVT, ANTI-DERIVATIVES, INDEFINITE INTEGRALS & INITIAL VALUE PROBLEMS



Math Bridge in Beijing

I (a) State *Rolle's Theorem*.

(b) State the *Mean Value Theorem*, and explain its geometric meaning.

(c) How is the MVT derived from Rolle's Theorem?

(d) Using the Mean Value Theorem, prove that if df/dx = dg/dx on (a, b), then there exists a constant *C* for which f(x) = g(x) + C for all $x \in (a,b)$.

(e) Let $f(x) = x^3 - 2x + 3$ be defined on the interval [1, 3]. Apply the MVT to this function and find the corresponding value of *c*.

(f) Let $g(x) = 1 + 3\sin 2x$ be defined on the interval $[0, \pi/12]$. Apply the MVT to this function and find the corresponding value of *c*.

II Evaluate each of the following *indefinite integrals* (using the method of "judicious guessing"):

$$(a)\ \int \frac{x^4 + x^3 + x + 1}{x}\, dx$$

$$(b)\ \int \frac{e^x}{1 + 4e^x}\, dx$$

$$(c)\ \int x^2 e^{4x^3}\, dx$$

$$(d)\ \int \frac{\sec^2 x}{1 + \tan x}\, dx$$

$$(e)\ \int \left(\frac{1}{x^2} + \frac{3}{x^2 + 1}\right) dx$$

$$(f)\ \int \ln x\, dx \quad (\textit{Try}\ \ x \ln x\ \textit{as a first guess.})$$

$$(g)\ \int \frac{\cos\left(\frac{1}{x}\right)}{x^2}\, dx$$

$$(h)\ \int x^2 (11x^3 + 99)^{51}\, dx$$

$$(i)\ \int t \sqrt[4]{1 + 2t^2}\, dt$$

$$(j)\ \int \frac{1}{(\arcsin z)\sqrt{1 - z^2}}\, dz$$

III Solve each of the following *differential equations* (using the method of "judicious guessing").

$$(a)\ \frac{dy}{dx} = \left(x + \frac{1}{x}\right)^2$$

$$(b)\ \frac{dy}{dx} = \sin^2 x \cos x$$

$$(c)\quad \frac{dy}{dx} = (1+3\ln x)\frac{1}{x}$$

$$(d)\quad \frac{d^2y}{dx^2} = \sinh x$$

$$(e)\quad \frac{dy}{dx} = \frac{\pi}{4}\sec^2\left(\frac{\pi}{4}x\right) - \frac{2\ln x}{x}$$

IV Solve each of the following *initial value problems* (using the method of "judicious guessing"):

$$(a)\quad \frac{dy}{dx} = 1 + x + \sin \pi x,\ \ y(0) = 5$$

$$(b)\quad \frac{dy}{dx} = \tan^2 x,\ \ y(0) = 7$$

$$(c)\quad \frac{dy}{dx} = \frac{x^2}{x^3+1} + x^3 + x + 7,\quad y(0) = 4$$

$$(d)\quad \frac{dy}{dx} = (x+5)\sqrt{x},\quad y(1) = 1$$

$$(e)\quad \frac{dy}{dx} = \frac{\sqrt{\ln x}}{x},\quad y(e) = 11$$

V Charlotte the spider is traveling along the x-axis with acceleration, a(t), given by:

$$a = \sqrt{t} - \frac{1}{\sqrt{t}}$$

Assume that at time t = 0 minute her velocity, v(0), is 4/3 cm/min and her position, x(0), is – 4/15 cm. Where is Charlotte at time t = 5 minutes?

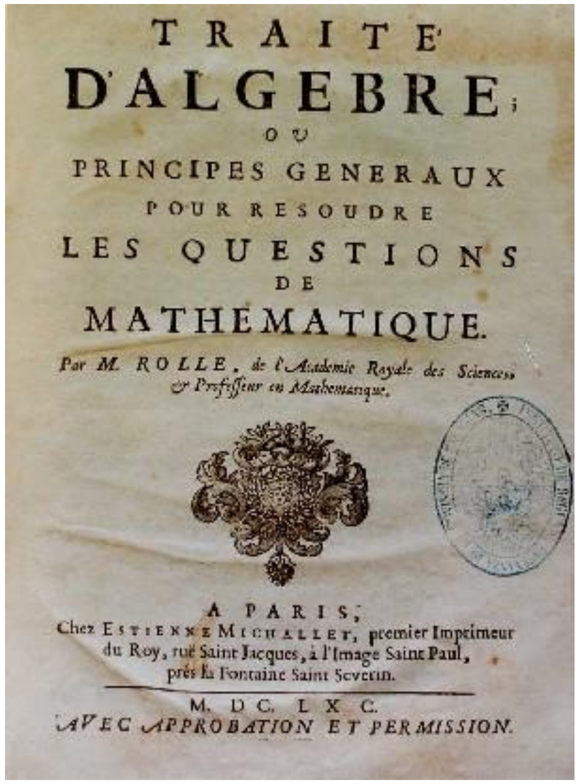
VI A grapefruit thrown upward has an initial velocity of 64 ft/sec from an initial height of 80 feet. (Recall that the acceleration due to gravity is -32 ft/sec^2.)

(a) Find the position, s(t), of the grapefruit as a function of time *t*.

(b) When does the grapefruit hit the ground?

VII Verify the following integration formula:

$$\int e^x \sin x \, dx = \frac{1}{2}\left(e^x \sin x - e^x \cos x\right) + C$$


TRAITE'

D'ALGEBRE;

OU

PRINCIPES GENERAUX

POUR RESOUDRE

LES QUESTIONS

DE

MATHEMATIQUE.

Par M. ROLLE, de l'Academie Royale des Sciences, & Professeur en Mathematique.

A PARIS,

Chez ESTIENNE MICHALLET, premier Imprimeur du Roy, ruë Saint Jacques, à l'Image Saint Paul, prés la Fontaine Saint Severin.

M. DC. LXC.

AVEC APPROBATION ET PERMISSION.

Michel Rolle (1652 –1719)