WORKSHEET XVII

THE RIEMANN INTEGRAL

Georg Friedrich Bernhard Riemann

(1826 – 1866)

1. Using the *area interpretation* of the Riemann integral, evaluate each of the following:

(a) $\int_{-2}^{1} |x|\,dx$

(b) $\int_{-3}^{2} |3x+4|\,dx$

(c) $\int_{0}^{1} \sqrt{1-x^2}\,dx$

(d) $\int_{-3}^{3} x^{1789}\sin(x^2+1)\,dx$

(e) $\int_{0}^{2\pi} \cos x\,dx$

2. Suppose that $\int_{-2}^{3}\left(f(x)+1\right)dx=0.$ Evaluate $\int_{-2}^{3}\left(f(x)-x\right)dx.$

3. Let *g* be a continuous function on the interval [-5, 5]. Suppose that

$$\int_{0}^{5} g(x)\,dx = 4$$

Evaluate each of the following Riemann integrals:

$(a)\ \int_{0}^{5}\left(g(x)+3\right)dx$

$(b)\ \int_{-2}^{3} g(x+2)\,dx$

$(c)\ \int_{-5}^{5} g(x)\,dx \ \ if\ g\ is\ even$

$(d)\ \int_{-5}^{5} g(x)\,dx \ \ if\ g\ is\ odd$

4. Find the constants *a* and *b* that *maximize* the value of the definite integral:

$$\int_{a}^{b}\left(4-x^{2}\right)dx$$

Justify your answer!

5. By using an appropriate Riemann sum, determine:

$$\lim_{n\to\infty}\frac{1}{n^{4}}\sum_{j=1}^{n} j^{3}$$

6. Find a formula for $\int_{a}^{b} x\,dx$

7. Express the *average value* of each of the following functions as a Riemann integral. *(Do not try to evaluate.)*

(a) f(x) = sin x over [0, π]

(b) $g(x) = (x-1)^2$ over [0, 3]

(c) h(x) = (ln x) / x over [1, 4]

(d) s(t) = cosh t over [0, ln 2]

8. State the major properties of the Riemann integral.

9. Suppose that *h* is integrable and that $\int_{-1}^{1} h(x)\,dx = 0$ *and* $\int_{-1}^{3} h(x)\,dx = 6.$

Find:

$(a)\ \int_{1}^{3} h(x)\,dx$

$(b)\ \int_{1}^{3} \left(5h(x)+3\right)\,dx$

10. Suppose that *f* and *h* are integrable and that

$$\int_{1}^{9} f(x)\,dx = -1,\ \int_{7}^{9} f(x)\,dx = 5 \text{ and } \int_{7}^{9} h(x)\,dx = 4$$

Find:

$(a)\ \int_{1}^{9} -3f(x)\,dx$

$(b)\ \int_{7}^{9} \left(f(x)+h(x)\right)\,dx$

$(c)\ \int_{7}^{9} (5f(x)-3h(x))\,dx$

$(d)\ \int_{1}^{7} (f(x)-|x-4|)\,dx$

11. Given the formula for $\int_{a}^{b} x^2\,dx$, find the *average value* of:

(a) $f(x) = x^2 - 1$ over [2, 4]

(b) $g(x) = (x - 2)^2$ over [0, 2]

(c) $h(x) = 5 - 3x - 4x^2$ over [0, 2]

I'm very good at integral and differential calculus,
I know the scientific names of beings animalculous;
In short, in matters vegetable, animal, and mineral,
I am the very model of a modern Major-General.
About binomial theorems I'm teeming with a lot of news,
With many cheerful facts about the square on the hypotenuse.

- W. S. Gilbert, **The Pirates of Penzance** (1879)