# WORKSHEET XVIII

## THE FTC

1. State the two versions of the *Fundamental Theorem of Calculus*.

2. Find the *area beneath the given curve lying above the given interval:*

   (a) $f(x) = x^3$ above $[1, 3]$

   (b) $g(x) = \sin x$ over $[0, \pi]$

   (c) $z(x) = (x - 1)^2$ over $[0, 3]$

   (d) $h(x) = (\ln x) / x$ over $[1, 4]$

   (e) $s(t) = t^3(2 + 3t^4)^3$ over $[0, 1]$

3. For each function in (2), find the *average value* over the given interval.

4. Using the FTC, compute $g'(e)$ given that

$$g(x) = \int_0^x t^5 (5 - 4\ln t)^{13}\, dt$$

5. Using the FTC compute:

$$\frac{d}{dx} \int_0^x e^{-u^2}\, du$$

6. Let $0 < k < 1$ and consider the Elliptic Integral:

$$E(x) = \int_0^x \frac{1}{\sqrt{1 - k^2 \sin^2 t}}\, dt$$

   Find $dE/dx$.

7. Using the FTC and the Chain Rule, calculate $dF/dx$ given that:

$$F(x) = \int_0^{\sin x} \frac{1}{1 + v^5}\, dv$$

8. Consider the function defined by:

$$H(x) = \int_{\frac{\pi}{2}}^{x^3} \cos t \, dt$$

Calculate dH/dt by:

(a) Using the FTC and the Chain Rule.

(b) By first performing the integration.

(c) Compare the two answers that you have obtained.

9. A particle is moving along a line so that its velocity is given by:

$v(t) = (t - 1)(t - 4)(t - 5) = t^3 - 10t^2 + 29t - 20$ ft/sec at time $t$ seconds.

(a) Find the *displacement* of the particle over the time interval [1, 5].

(b) Find the *total distance* traveled by the particle over the time interval [1, 5].

10. Find the *area* of the region bounded by the graphs of the given functions. Sketch!

(a) $y = x^2 + 2$, $y = -x$, $x = 0$, and $x = 1$.

(b) $y = 2 - x^2$ and $y = x$

(c) $y = \sin x$ and $y = \cos x$ over $[\pi/4, 5\pi/4]$

(d) $y = 3x^3 - x^2 - 10x$ and $y = 2x - x^2$

(e) $y = x^3$ and $y = x^6$

(f) $y = x^3 - x$ and $y = 0$

(g) $y = x^2 - 4x + 3$ and $y = 3 + 2x - x^2$