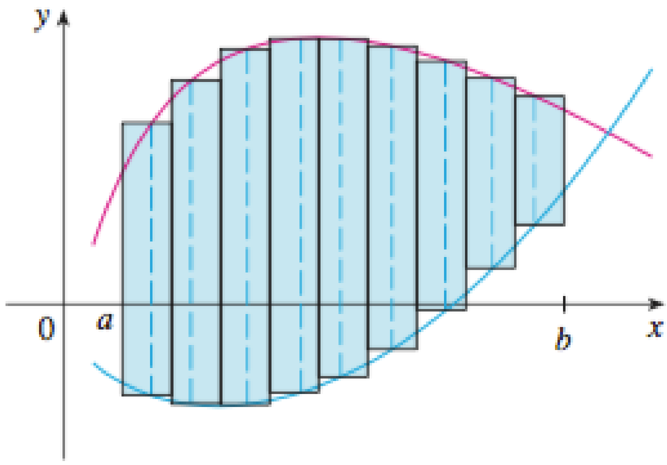
WORKSHEET XIX

AREA BOUNDED BY CURVES



1. Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.
2. Find the area of the region in the first quadrant bounded above by $y = x^{1/2}$ and below by the x-axis and the line $y = x - 2$.
3. Repeat exercise (2) above, but this time integrate with respect to *y*.
4. Find the area of the crescent-shaped region in the first quadrant that is bounded by $y = x^{13}$ and $y = x^{15}$.
5. Find the area of the region bounded by $y = 7 - 2x^2$ and $y = x^2 + 4$.
6. Find the area of the region enclosed by $y = x^4 - 4x^2 + 4$ and $y = x^2$.
7. Find the area of the region enclosed by $y = x^4 - 4x^2 + 4$ and $y = x^2$.
8. Find the area of the region enclosed by $y = x(a^2 - x^2)^{1/2}$, where $a > 0$, and $y = 0$.
9. Find the area of the region enclosed by $y = (|x|)^{1/2}$ and $5y = x + 6$.
10. Find the area of the region enclosed by $x = y^3 - y^2$ and $x = 2y$.
11. Find the area of the region bounded by $4x^2 + y = 4$ and $x^4 - y = 1$.
12. Find the area of the region enclosed by $y = 2 \sin x$ and $y = \sin(2x)$, $0 \le x \le \pi$.
13. Find the area of the region enclosed by $y = \cos(\pi x/2)$ and $y = 1 - x^2$.
14. Find the area of the region enclosed by $y = \sin(\pi x/2)$ and $y = x$.
15. Find the area of the "triangular" region in the first quadrant that is bounded above by the curve $y = e^{2x}$, below by the curve $y = e^x$, and on the right by the line $x = \ln 3$.

DIFFERENTIATING INTEGRALS

Differentiate with respect to x each of the following integrals using the FTC and Leibniz's Formula:

1. $y = \int_{3}^{x} \sqrt{5 + \cos^3 t}\, dt$

2. $y = \int_{1}^{x} \frac{5}{3 + t^4}\, dt$

3. $y = \int_{\sec x}^{4} \frac{1}{1 + t^2}\, dt$

4. $y = \int_{1/x}^{x} \frac{1}{t}\, dt$

5. $y = \int_{\cos x}^{\sin x} \frac{1}{1 - t^2}\, dt$

6. $y = \int_{\sqrt{x}}^{x^2} \frac{e^t}{t}\, dt$

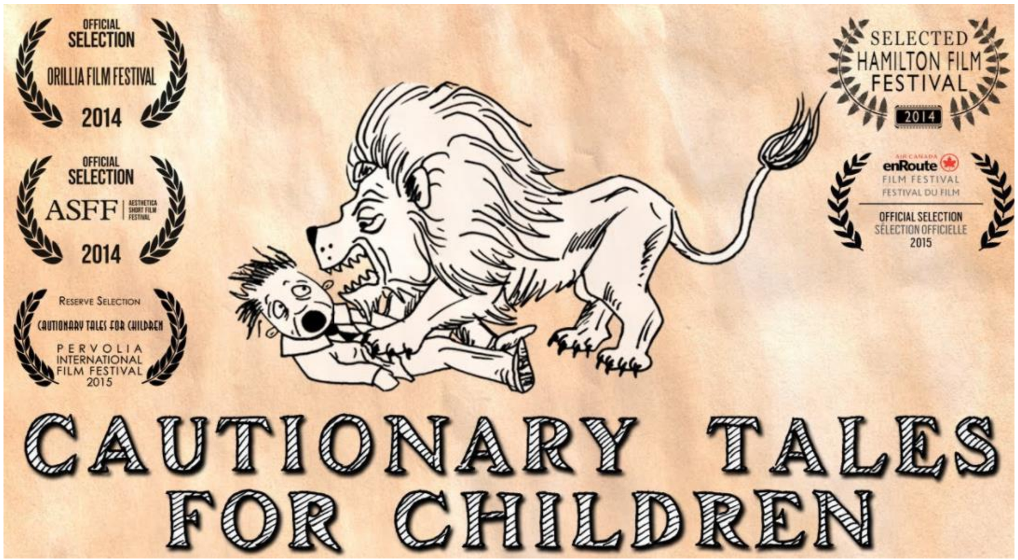
USING INTEGRALS TO APPROXIMATE RIEMANN SUMS

Evaluate each of the following limits:

1. $\lim_{n \to \infty} \frac{1^5 + 2^5 + 3^5 + \ldots + n^5}{n^6}$

2. $\lim_{n \to \infty} \frac{1^3 + 2^3 + 3^3 + \ldots + n^3}{n^4}$

3. $\lim_{n \to \infty} \frac{1}{n} \left(\sin\frac{\pi}{n} + \sin\frac{2\pi}{n} + \sin\frac{3\pi}{n} + \ldots + \sin\frac{n\pi}{n} \right)$



The nicest child I ever knew
Was Charles Augustus Fortescue.
He never lost his cap, or tore
His stockings or his pinafore:
In eating Bread he made no Crumbs,
He was extremely fond of sums.

\- Hilaire Belloc, **Cautionary Tales** (1907)