WORKSHEET XX

L'HÔPITAL'S RULE

Marquis Guillaume de l'Hôpital (1661 – 1704)

I Evaluate each of the following limits, using l'Hôpital's rule when appropriate:

(A) $\displaystyle\lim_{x\to 0} \frac{e^{3x}-1}{x}$

(B) $\displaystyle\lim_{x\to\infty} \frac{x^2}{e^x}$

(C) $\displaystyle\lim_{x\to\infty} e^{-x}\sqrt{x}$

(D) $\displaystyle\lim_{x\to\infty} \left(1+\frac{1}{x}\right)^x$

(E) $\displaystyle\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x}$

(F) $\displaystyle\lim_{x\to\infty} x^{1/x}$

(G) $\displaystyle\lim_{x\to 0} \frac{\sqrt{1+x}-1-x/2}{x^2}$

(H) $\displaystyle\lim_{x\to 0} \frac{e^x-1-x}{x^2}$

(I) $\displaystyle\lim_{x\to 0} \frac{\cos x-1+x^2/2}{x^4}$

(J) $\displaystyle\lim_{x\to 0} \frac{x(\cosh x-1)}{\sinh x-x}$

(K) $\displaystyle\lim_{x\to\infty} \frac{\ln(ax+b)}{\ln(cx+d)}$ (where a, b, c, d are positive constants)

(L) $\displaystyle\lim_{x\to 1} \frac{x^{2017}-1}{x-1}$

(M) $\displaystyle\lim_{x\to\pi/2-} \frac{\tan x}{\sec x+1}$

(N) $\displaystyle\lim_{x\to 0+} (\sin x)^x$

(O) $\displaystyle\lim_{x\to 1} \frac{\ln x}{x-1}$

(P) $\displaystyle\lim_{x\to 0+} x\ln x$

(Q) $\displaystyle\lim_{x\to\infty} \left(1+\frac{5}{x}\right)^x$

(R) $\displaystyle\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$

(S) $\displaystyle\lim_{x \to 1+} x^{\frac{1}{x-1}}$

(T) $\displaystyle\lim_{x \to \infty} \frac{\displaystyle\int_{1}^{x} \frac{t^4 + t + 1}{t^2 + 4t + 13 \ln t} \, dt}{(2x+1)^3}$

(U) $\displaystyle\lim_{x \to 1+} \left(\frac{1}{\ln x} - \frac{1}{x-1} \right)$

(V) Explain what happens if one tries to use l'Hopital's rule on the

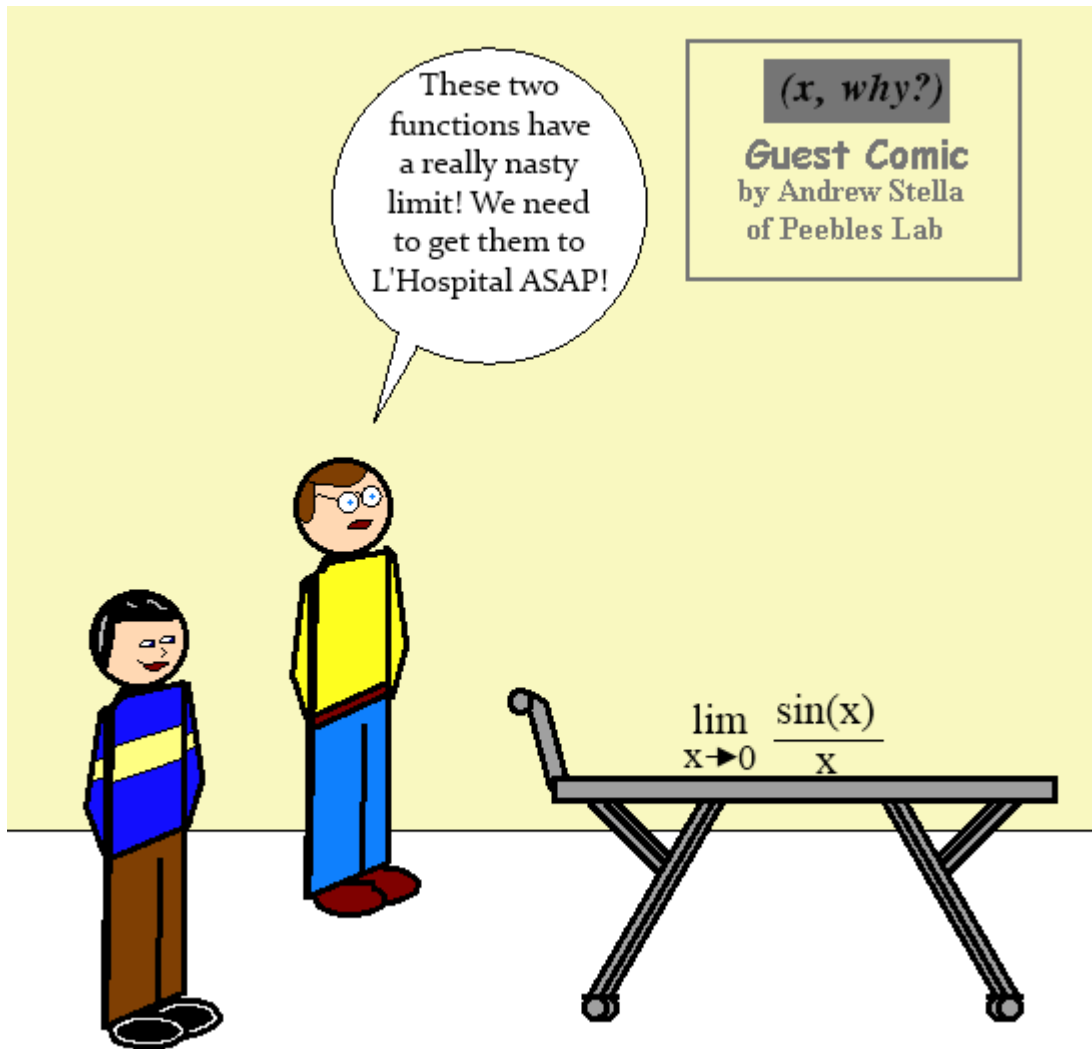following: $\displaystyle\lim_{x \to \infty} \frac{x}{\sqrt{x^2 - 1}}$

II For each of the following, find the value for c that makes the function continuous at x = 0.

(A) $f(x) = \begin{cases} \dfrac{4x - 2 \sin 2x}{2x^3} & \text{if } x \neq 0 \\[4mm] c \text{ if } x = 0 \end{cases}$

(B) $g(x) = \begin{cases} \left(e^x + x \right)^{1/x} & \text{if } x \neq 0 \\[4mm] c \text{ if } x = 0 \end{cases}$

III The *Gamma Function*, $\Gamma(n)$, is defined in terms of the integral of the function $f(x) = x^{n-1}e^{-x}$ for $n > 0$. Show that for any fixed value of n, the limit of $f(x)$ as $x \to \infty$ is 0.

The notion of infinity is our greatest friend; it is also the greatest enemy of our piece of mind.

- James Pierpont