**WORKSHEET XXI**

**Newton’S Method**



1. Using *Newton’s method*, estimate the positive solution to x2 – 3 = 0. Note that the Intermediate Value Theorem guarantees the existence of such a solution. (Why?)

Start with an initial guess of x0 = 2. (Of course, we know the exact answer before we begin the process, but we can better appreciate how quickly Newton’s method converges to the root.)

1. Using Newton’s method, estimate the solutions to the equation x2 + x – 1 = 0. Start with x0 = -1 for the solution on the left and x0 = 1 for the solution on the right. In each case, find x2.
2. Use Newton’s method to estimate the one real solution of x3 + 3x + 1 = 0. Begin with x0 = 0 and then find x2. (Explain why there is only one real root.)
3. Find a solution to the equation x = 1 + 0.5 sin x using Newton’s method. Graphing would suggest that there is a solution near x = 1.5.
4. Let G(x) = x4 – 3x3 + 4x – 1. Walt wants to find a root of the equation G(x) = 0. First he observes that G(0) < 0 and G(1) > 0. Then letting x0 = 0.5, Walt employs Newton’s method to find better approximations to the root between x = 0 and x = 1. Find x1 and x2 (each to 5 significant digits). Show your work!



*When I am working on a problem I never think about beauty. I only think about how to solve the problem. But when I have finished, if the solution is not beautiful, I know it is wrong.*

- Buckminster Fuller (1895-1983)