WORKSHEET VI

SHORTCUTS

"Don't worry... I know a short cut."

I   Using the short cuts of differentiation *when appropriate*, compute the derivative of each of the following functions.

(A) $y = 2017 + 3x - \pi x^4 + e^4$

(B) $y = x \sin x$

(C) $y = \dfrac{x+3}{x+7}$

(D) $y = \dfrac{x}{\sin x}$

(E) $y = \dfrac{\cos x}{x^3 + 9}$

(F) $y = (x^2 + 4x - 1)(x^3 + 5x^4 - x^3 + x^2 + 3x + 13)$

(G) $y = \sin^2 x$

(H) $y = (x^2 + 5x + 1)^2$

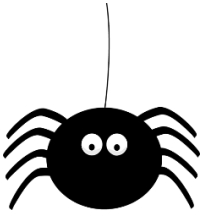II   (a) Find the equations of the *tangent* and *normal lines* to the curve

$$y = \frac{x - 4}{x + 1} \ \ at \ x = 3.$$

(b) Find the equations of the *tangent* and *normal* lines to the curve

$$y = \sin x \ at \ x = \pi/4.$$

III   Using appropriate shortcuts, find formulas for the derivatives of

$$y = \tan x \ \ and \ \ y = \sec x.$$

IV   Charlotte, the spider, dances along the x-axis according to the rule
$x(t) = t^3 - 3t + 5$. (Here time is measured in *seconds* and distance in *cm.*)

(a) Find Charlotte's *velocity* at time t = 2 sec.

(b) Find Charlotte's *acceleration* at time t = 2 sec.

V   Sketch the curve $y = x^2(x - 2)^2$. Over which interval(s) is the graph *rising? falling?*   Locate any local maxima or minima.

VI   Sketch the curve $y = \frac{x-4}{x+1}$ (cf. problem II a). Over which interval(s) is the graph *rising? falling?*   Locate any *local maxima* or *minima*.

VII   Sketch the curve $y = xe^x$. Over which interval(s) is the graph rising? falling?   Locate any local maxima or minima.

VIII   Sketch the curve $y = \frac{x-3}{x^2+1}$. Over which interval(s) is the graph rising? falling?   Locate any local maxima or minima.

IX   Consider the curve $y = b + c \sin x$.   For each of the following values of b and c, determine when the graph is rising and when it is falling:

(1) b = 3, c = 1
(2) b = c = 1
(3) b = 1, c = 2

X   Sketch the curve $y = 1/x + x^2$ over the interval $(0, \infty)$. Over which interval(s) is the graph *rising? falling?*   Locate any local maxima or minima.

What Romantic terminology called genius or talent or inspiration is nothing other

than finding the right road empirically, following one's nose, taking shortcuts.

-    Italo Calvino (1923 – 1985)