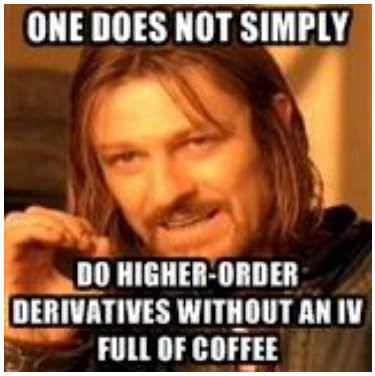
WORKSHEET VII

HIGHER-ORDER DERIVATIVES



1. Find the first *three* derivatives of each of the following functions.

$(A)\quad y = ax^2 + bx + c$

$(B)\quad y = 2x^3 + \dfrac{1}{x^2} + e^x$

$(C)\quad y = xe^x$

$(D)\quad y = \sin x$

$(E)\quad y = x \sin x$

$(F)\quad y = x^2 \ln x$

2. (A) If $(d/dx)e^{4x} = 4e^{4x}$, find $(d^{199}/dx^{199})\, e^{4x}$.

(B) If $(d/dx) \sin 5x = 5 \cos 5x$, and $(d/dx) \cos 5x = -5 \sin 5x$, find $(d^{2017}/dx^{2017}) \sin 5x$.

3. If $f(x) = x^{1/2}$, find $f^{(4)}(x)$.

4. If $x(t) = 3t^3 - 4t + 1$ is the position (measured in meters) of Charlotte on the x-axis at time t (measured in hours), find Charlotte's *velocity* and *acceleration* at time t = 2 hrs.

5. If $F(x) = x^m$, find $F^{(m)}(x)$. (Assume that m is a positive integer.)

6. Let y = ln x. Given that dy/dx = 1/x, find d^4y/dx^4. Can you find $d^{10}y/dx^{10}$?

7. *(University of Michigan)* Consider the following table giving values, rounded to three decimal places, of a function $f(x)$.

x	0	0.5	1
$f(x)$	0	0.247	0.841

a) Estimate $f'(1)$. Be sure it is clear how you obtain your answer.

b) Estimate $f''(1)$. Again, be sure that it is clear how you obtain your answer.

c) Estimate $f(1.25)$ being sure your work is clear.

d) Based on your work in (a) and (b), is your estimate in (c) an over- or underestimate? Explain.

8. *(University of Michigan)* A paperback book (definitely not a valuable calculus textbook, of course) is dropped from the top of Mertz hall (which is 40 m high) towards a very large, upward pointing fan. The average velocity of the book between time $t = 0$ and later times is shown in the table of data below (in which t is in seconds and the velocities are in m/s).

between $t = 0$ seconds and $t =$	1	2	3	4	5
the average velocity is	−5	−10	−11.67	−9	−7.2

a) Fill in the following table of values for the height $h(t)$ of the book (measured in meters). Show how you obtain your values.

t	0	1	2	3	4	5
$h(t)$	40	_____	_____	_____	_____	_____

b) Based on your work from (a), is $h''(1) > 0$, < 0, or $= 0$? Is $h''(3) > 0$, < 0, or $= 0$? Explain.

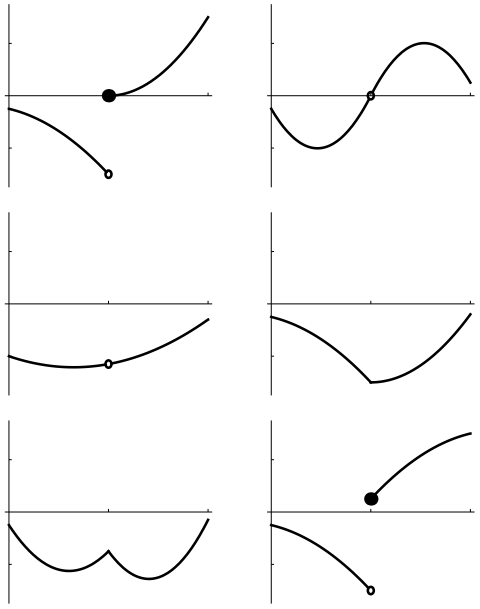
9. For each of the descriptions of a function f that follow, indicate which of the graphs below match the description. For each description there may be no, one, or several graphs that match; write **none** if no graphs match the description. You may need to use a graph more than once. In each case you should assume that f is defined only on the domain [0, 2].

a) $f''(x) < 0$ for $x < 1$ and $f''(x) > 0$ for $x > 1$; $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$; and $f(x)$ is continuous everywhere except at $x = 1$.

b) $f''(x) > 0$ for all $x < 1$; $f''(x) < 0$ for all $x > 1$; and $f(x)$ is differentiable everywhere except at $x = 1$.

c) $f''(x) < 0$ for all $x < 1$; $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$; and $f(x) < 0$ for all $x = 1$.

d) $f''(x) < 0$ for $x < 1$ and $f''(x) > 0$ for $x > 1$; $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$; and $f(x)$ is differentiable everywhere except at $x = 1$.



10.

The graph of a function f is given in Figure 3.1. If f is a polynomial of degree 3, then the value of $f'''(0)$ is

(a) Positive
(b) Negative
(c) Zero

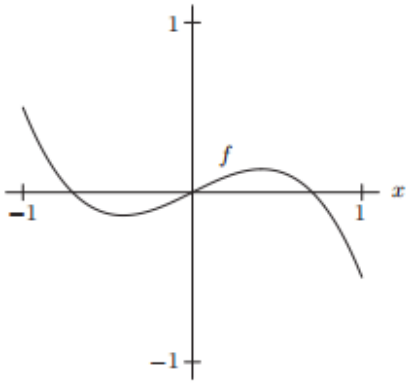


Figure 3.1

11.

The graph of a function f is given in Figure 3.2. If f is a polynomial of degree 3, then $f'''(0)$ is

(a) Positive
(b) Negative
(c) Zero

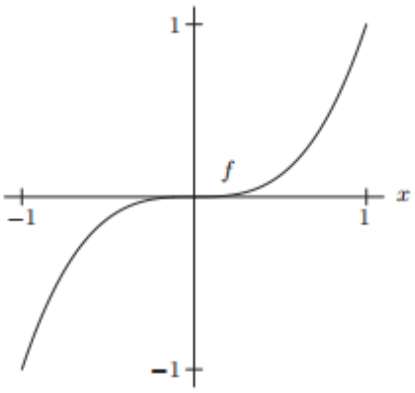


Figure 3.2

12.

The graph of a function f is given in Figure 3.5. If f is a polynomial of degree 3, then the values of $f'(0)$, $f''(0)$, and $f'''(0)$ are (respectively)

(a) $-, +, +$ (b) $-, -, -$ (c) $-, +, -$
(d) $-, +, +$ (e) $+, -, +$ (f) $+, +, +$

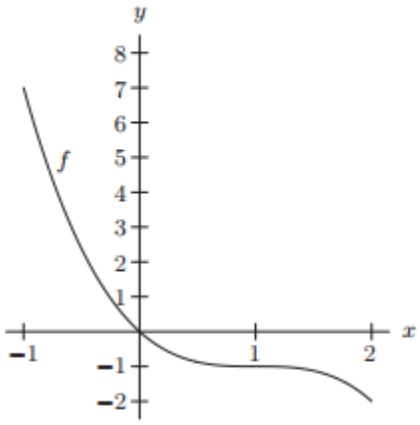


Figure 3.5

13.

The graph of a function f is given in Figure 3.6. If f is a polynomial of degree 3, then the values of $f'(0)$, $f''(0)$, and $f'''(0)$ are (respectively)

(a) $-, -, +$ (b) $-, 0, -$ (c) $-, +, -$
(d) $-, +, +$ (e) $+, -, +$ (f) $+, +, +$

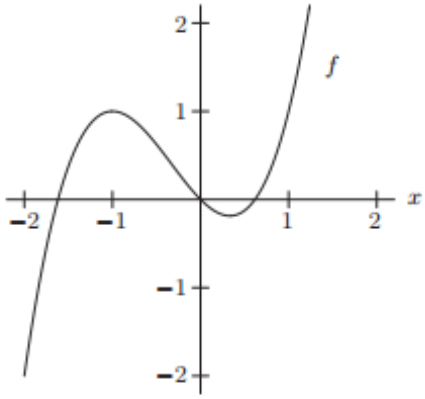


Figure 3.6