WORKSHEET VIII

CURVE SKETCHING: A PRELUDE (REVISED)

1.

If the graph in Figure 4.3 is that of $f'(x)$, which of the following statements is true concerning the function f?

(a) The derivative is zero at two values of x, both being local maxima.
(b) The derivative is zero at two values of x, one is a local maximum while the other is a local minimum.
(c) The derivative is zero at two values of x, one is a local maximum on the interval while the other is neither a local maximum nor a minimum.
(d) The derivative is zero at two values of x, one is a local minimum on the interval while the other is neither a local maximum nor a minimum.
(e) The derivative is zero only at one value of x where it is a local minimum.

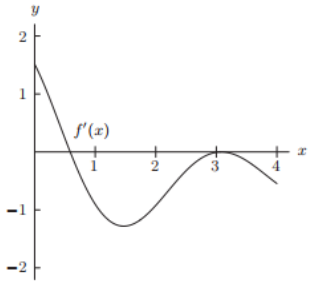


Figure 4.3

2.

Concerning the graph of the function in Figure 4.2, which of the following statements is true?

(a) The derivative is zero at two values of x, both being local maxima.
(b) The derivative is zero at two values of x, one is a local maximum while the other is a local minimum.
(c) The derivative is zero at two values of x, one is a local maximum on the interval while the other is neither a local maximum nor a minimum.
(d) The derivative is zero at two values of x, one is a local minimum on the interval while the other is neither a local maximum nor a minimum.
(e) The derivative is zero only at one value of x where it is a local minimum.

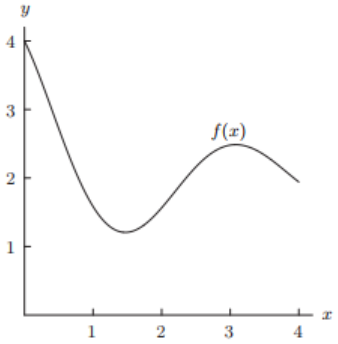


Figure 4.2

3. Consider the graph of the function below.

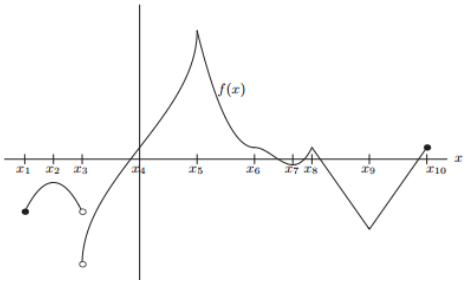
(a) How many critical points does f have?

(b) How many local minima does f have? Where are they located?

(c) How many local maxima does f have? Where are they located?

(d) Where is the global max of f(x)?

(e) Where is the global min of f(x)?



4. State the *Compactness Theorem* (aka *Extreme Value Theorem*).

5. *True or False?* A global max is always a critical point.

6. *True or False?* A function defined on a closed interval [a, b] must have a global maximum *and* a global minimum.

7.
[10 points] The cable of a suspension bridge with two supports $2L$ meters apart hangs H meters above the ground. The height H is given in terms of the distance in meters from the first support x (in meters) by the function

$$H(x) = e^{x-L} + e^{L-x} + H_0 - 2$$

where H_0 and L are positive constants. Notice that x ranges from 0 (the first support) to $2L$ (the second support).

a. [4 points] Find (but do not classify) the critical points for the function $H(x)$.

b. [6 points] Find the x and y coordinates of all global maxima and minima for the function $H(x)$. Justify your answers.

8. Find the critical points of the function $f(x) = (x - 3)^5(x + 4)^9$. Sketch the curve.

Stewart exercises:

15. $f(x) = \frac{1}{2}(3x - 1), \quad x \leq 3$

16. $f(x) = 2 - \frac{1}{3}x, \quad x \geq -2$

17. $f(x) = 1/x, \quad x \geq 1$

18. $f(x) = 1/x, \quad 1 < x < 3$

19. $f(x) = \sin x, \quad 0 \leq x < \pi/2$

20. $f(x) = \sin x, \quad 0 < x \leq \pi/2$

21. $f(x) = \sin x, \quad -\pi/2 \leq x \leq \pi/2$

22. $f(t) = \cos t, \quad -3\pi/2 \leq t \leq 3\pi/2$

23. $f(x) = \ln x, \quad 0 < x \leq 2$

24. $f(x) = |x|$

25. $f(x) = 1 - \sqrt{x}$

26. $f(x) = e^x$

27. $f(x) = \begin{cases} x^2 & \text{if } -1 \leq x \leq 0 \\ 2 - 3x & \text{if } 0 < x \leq 1 \end{cases}$

28. $f(x) = \begin{cases} 2x + 1 & \text{if } 0 \leq x < 1 \\ 4 - 2x & \text{if } 1 \leq x \leq 3 \end{cases}$

Stewart exercises:

Find the critical points of each function defined below:

29. $f(x) = 4 + \frac{1}{3}x - \frac{1}{2}x^2$

30. $f(x) = x^3 + 6x^2 - 15x$

31. $f(x) = 2x^3 - 3x^2 - 36x$

32. $f(x) = 2x^3 + x^2 + 2x$

33. $g(t) = t^4 + t^3 + t^2 + 1$

34. $g(t) = |3t - 4|$

35. $g(y) = \dfrac{y - 1}{y^2 - y + 1}$

36. $h(p) = \dfrac{p - 1}{p^2 + 4}$

37. $h(t) = t^{3/4} - 2t^{1/4}$

38. $g(x) = \sqrt[3]{4 - x^2}$

39. $F(x) = x^{4/5}(x - 4)^2$

40. $g(\theta) = 4\theta - \tan\theta$

41. $f(\theta) = 2\cos\theta + \sin^2\theta$

42. $h(t) = 3t - \arcsin t$

43. $f(x) = x^2 e^{-3x}$

44. $f(x) = x^{-2}\ln x$

Find the global extrema of each of the following functions:

47. $f(x) = 12 + 4x - x^2, \quad [0, 5]$

48. $f(x) = 5 + 54x - 2x^3, \quad [0, 4]$

49. $f(x) = 2x^3 - 3x^2 - 12x + 1, \quad [-2, 3]$

50. $f(x) = x^3 - 6x^2 + 5, \quad [-3, 5]$

51. $f(x) = 3x^4 - 4x^3 - 12x^2 + 1, \quad [-2, 3]$

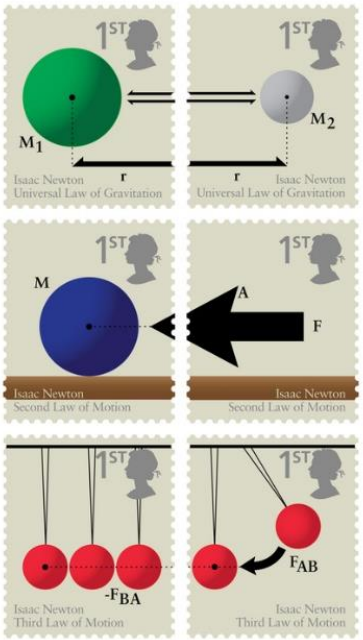
52. $f(t) = (t^2 - 4)^3, \quad [-2, 3]$

53. $f(x) = x + \dfrac{1}{x}, \quad [0.2, 4]$

54. $f(x) = \dfrac{x}{x^2 - x + 1}, \quad [0, 3]$

55. $f(t) = t - \sqrt[3]{t}, \quad [-1, 4]$

56. $f(t) = \dfrac{\sqrt{t}}{1 + t^2}, \quad [0, 2]$



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