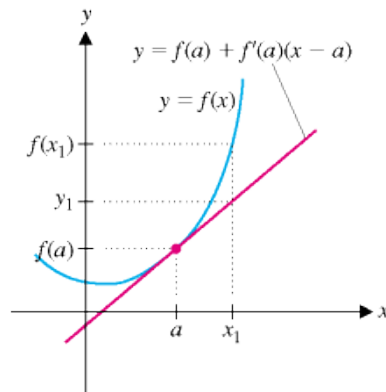# WORKSHEET IX

LINEAR APPROXIMATIONS

1. Find the linearization of the function $f(x) = \sqrt{x+3}$ at the point $x = 1$ and use it to approximate $\sqrt{3.98}$ and $\sqrt{4.05}$. For each approximation, is it an underestimate or an overestimate? Explain. (Here you may use the power rule short cut.)

2. Find the linearization of the function $f(x) = \sin x$ at the point $x = \pi/6$.

3. Find the linearization of the function $f(x) = (1+x)^{-3}$ at the point $x = 0$ and use it to approximate the value of $\frac{1}{1.003^3}$. *Is your* approximation an underestimate or an overestimate? Explain.

4. *(U. Michigan)* Given below is the graph of a function h(t). Suppose j(t) is the local linearization of h(t) at $t = 7/8$.
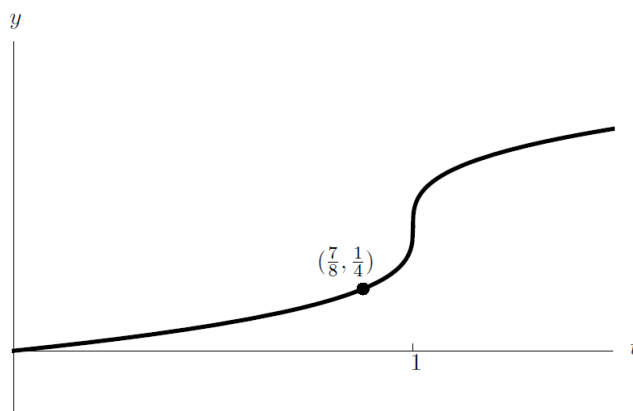
   (a)   Given that $h'\left(\frac{7}{8}\right) = \frac{2}{3}, find\ an\ expression$ for j(t).

   (b) Use your answer from (a) to approximate h(1).

   (c) Is the approximation from (b) an over- or under-estimate? Explain.

   (d) Using j(t) to estimate values of h(t), will the estimate be more accurate at t=1 or t = ¾? Explain.
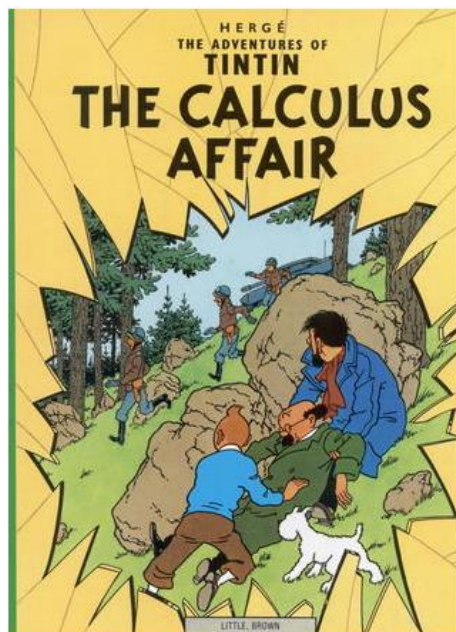
5. *(U. Michigan)*

[12 points] In Srebmun Foyoj, Maddy and Cal are eating lava cake. Let $T(v)$ be the time (in seconds) it takes Maddy to eat a v cm^3 serving of lava cake. Assume $T(v)$ is invertible and differentiable for $0 < v < 1000$. Several values of $T(v)$ and its first and second derivatives are given in the table below.

v	10	15	60	100	150	200	300
$T(v)$	11	22	84	194	393	513	912
$T'(v)$	2.4	1.9	1.8	3.6	3.7	0.9	17.5
$T''(v)$	-0.11	-0.08	0.05	0.04	-0.04	-0.05	0.59

Remember to show your work carefully.

a. [4 points] Use an appropriate linear approximation to estimate the amount of time it takes Maddy to eat a 64 cm^3 serving of lava cake. *Include units.*

b. [4 points] Use the quadratic approximation of $T(v)$ at $v = 200$ to estimate $T(205)$. (Recall that a formula for the quadratic approximation $Q(x)$ of a function $f(x)$ at $x = a$ is $Q(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2}(x - a)^2$.)

c. [4 points] Let $C(v)$ be the time (in seconds) it takes Cal to eat a v cm^3 serving of lava cake, and suppose $C(v) = T(\sqrt{v})$. Let $L(v)$ be the local linearization of $C(v)$ at $v = 100$. Find a formula for $L(v)$. Your answer should <u>not</u> include the function names T or C.

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