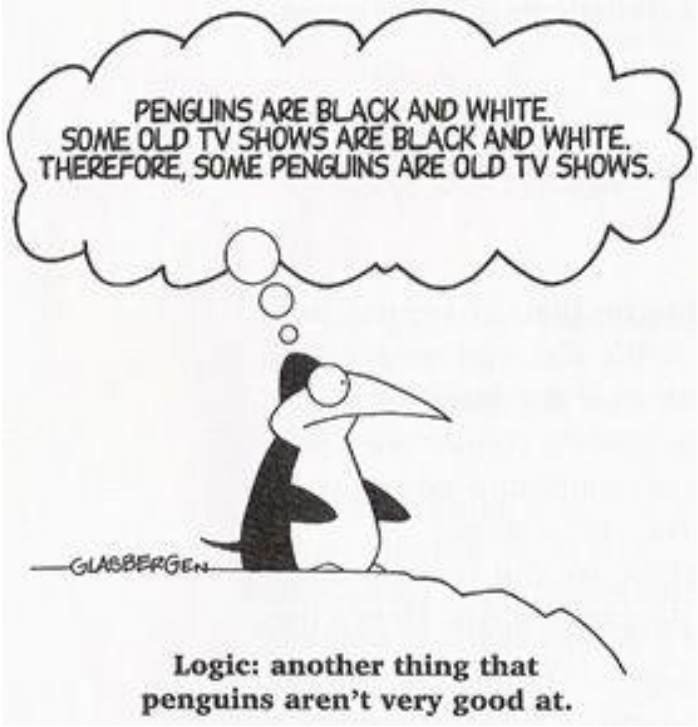
CLASS DISCUSSION: 29 AUGUST 2018

(CALCULATOR FREE)



1. *(algebra review)* For which non-zero value of *k* will the following quadratic equation have *only one* real root?

$$7x^2 + kx + 3k = 0$$

2. Consider the following rational functions. For each function, determine the limiting behavior as $x \to \infty$. Briefly explain how you arrived at your answers.

(a) $y = \dfrac{(2x-5)^2}{x^2}$

Answer: As $x \to \infty$, $y \to$ _____

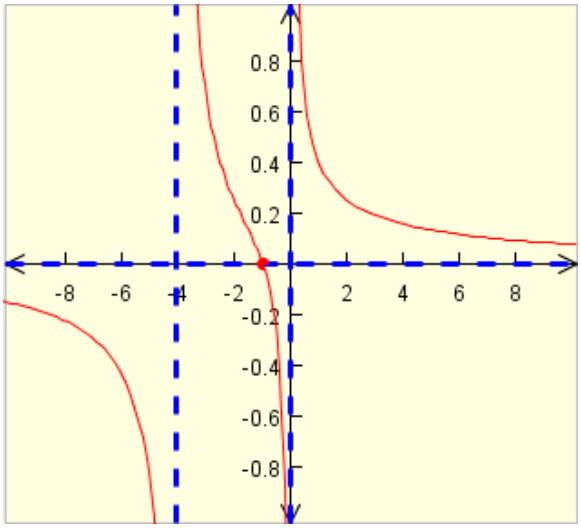
(b) $y = \dfrac{(x+5)^2(x-11)^3}{(x-9)(x+13)(x-1)(7x-44)}$

Answer: As $x \to \infty$, $y \to$ _____

(c) $y = \dfrac{99(x-15)(3x+11)}{(x-9)^2(19x+13)}$

Answer: As $x \to \infty$, $y \to$ _____

3. Find an *equation* of a rational function whose graph is given below:



4. Sketch the graph of each of the following rational functions (that includes all the significant properties):

$(a)\ f(x)=(x-1)^2(x-3)^3(3x+5)$

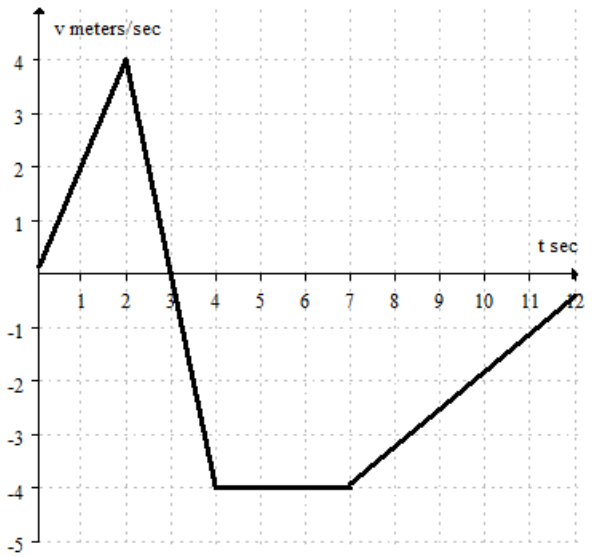
$(b)\ f(x)=\dfrac{x^2+5}{(x+1)^2}$

$(c)\ f(x)=\dfrac{(x-1)(x+2)}{x-3}$

$(d)\ f(x)=\dfrac{(x-3)^2(x-4)^3(x-5)^5}{x^2(2x^2+x+1)^4}$

5. (*University of Michigan* problem)

The graph below shows the velocity of a bug traveling along a straight line on the classroom floor.



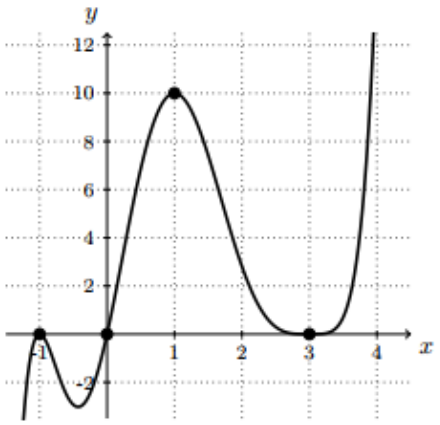
At what time(s) does the bug turn around?

A) At 3 seconds.
B) At 2 seconds and again at 7 seconds.
C) At 4 seconds and again at 7 seconds.
D) Never.

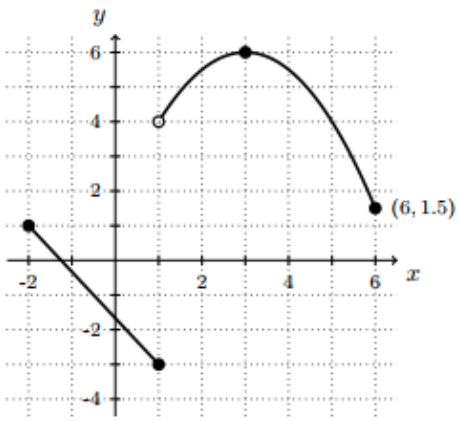
6. (*University of Michigan* problem)

[11 points] Consider the graphs of $y = k(x)$ and $y = \ell(x)$ given below:

GRAPH OF $y = k(x)$



GRAPH OF $y = \ell(x)$



You must **show your work** in both parts of this problem to receive full credit. Write your final answers *in the spaces provided.*

a. [5 points] Find a formula for $k(x)$, assuming $k(x)$ is a polynomial of degree seven with zeros at $x = -1$, $x = 0$ and $x = 3$.

b. [6 points] Find a piecewise-defined formula for $\ell(x)$ on $[-2, 6]$, given that the graph of $y = \ell(x)$ is made up of a line and a parabola.

7. Compute each of the following limits. Explain your reasoning. Do not use calculators.

(a) $\lim_{x\to\infty} \dfrac{(x+11)^2(3x-7)^3}{(2x^2+4)^4(x+2017)}$

(b) $\lim_{x\to\infty} \dfrac{1+\sqrt{x}}{5+x^2}$

8. Consider the rational function *F* defined by

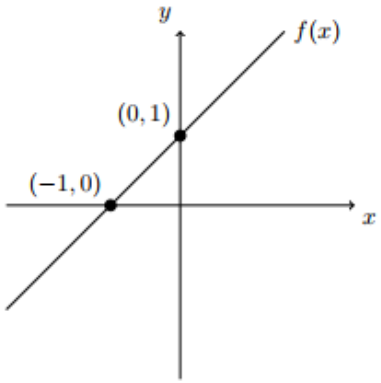
$$F(x)=\frac{15x^3+x^2-6x}{6x^2+x-2} \text{ if } x\neq 1/2 \text{ and } x\neq -2/3$$

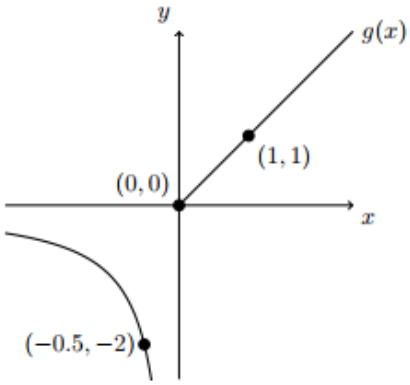
(a) Find the lim F(x) as x → ∞ if it exists. Explain.

(b) Find the lim F(x) as x → -∞ exist? Explain.

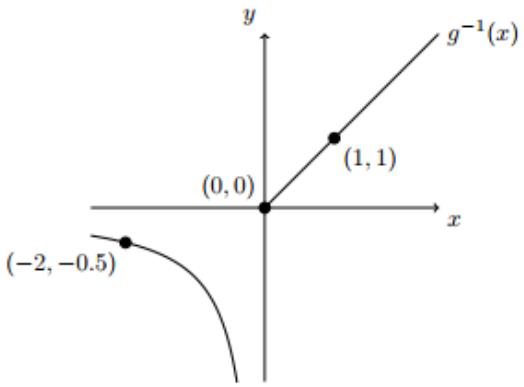
9. (*University of Michigan* problem

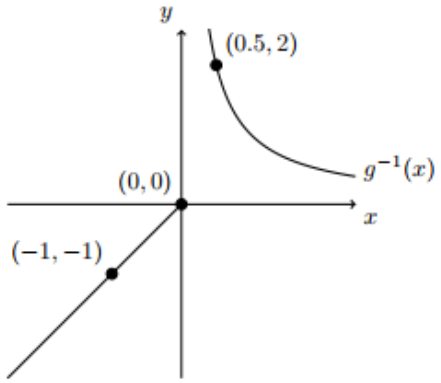
[5 points] A portion of the graphs of $y = f(x)$ and $y = g(x)$ are given below. **You do not need to show any work for this problem.**

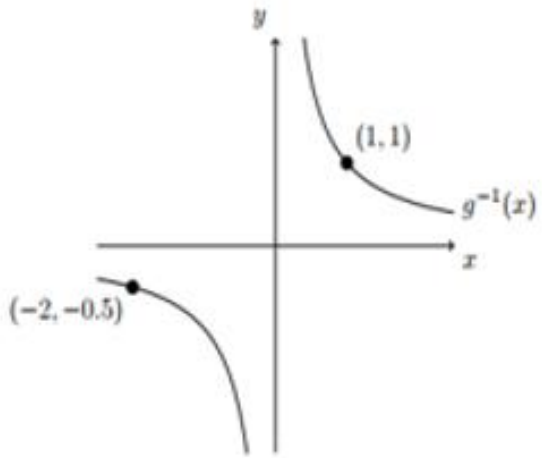


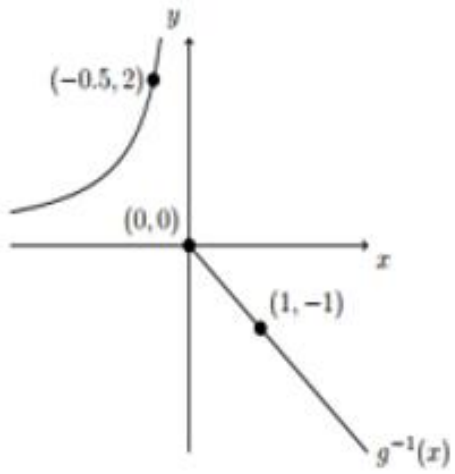


a. [2 points] Assume that $g(x)$ is an invertible function. Which of the following could be the graph of $y = g^{-1}(x)$? **Circle exactly one of the four graphs below.**

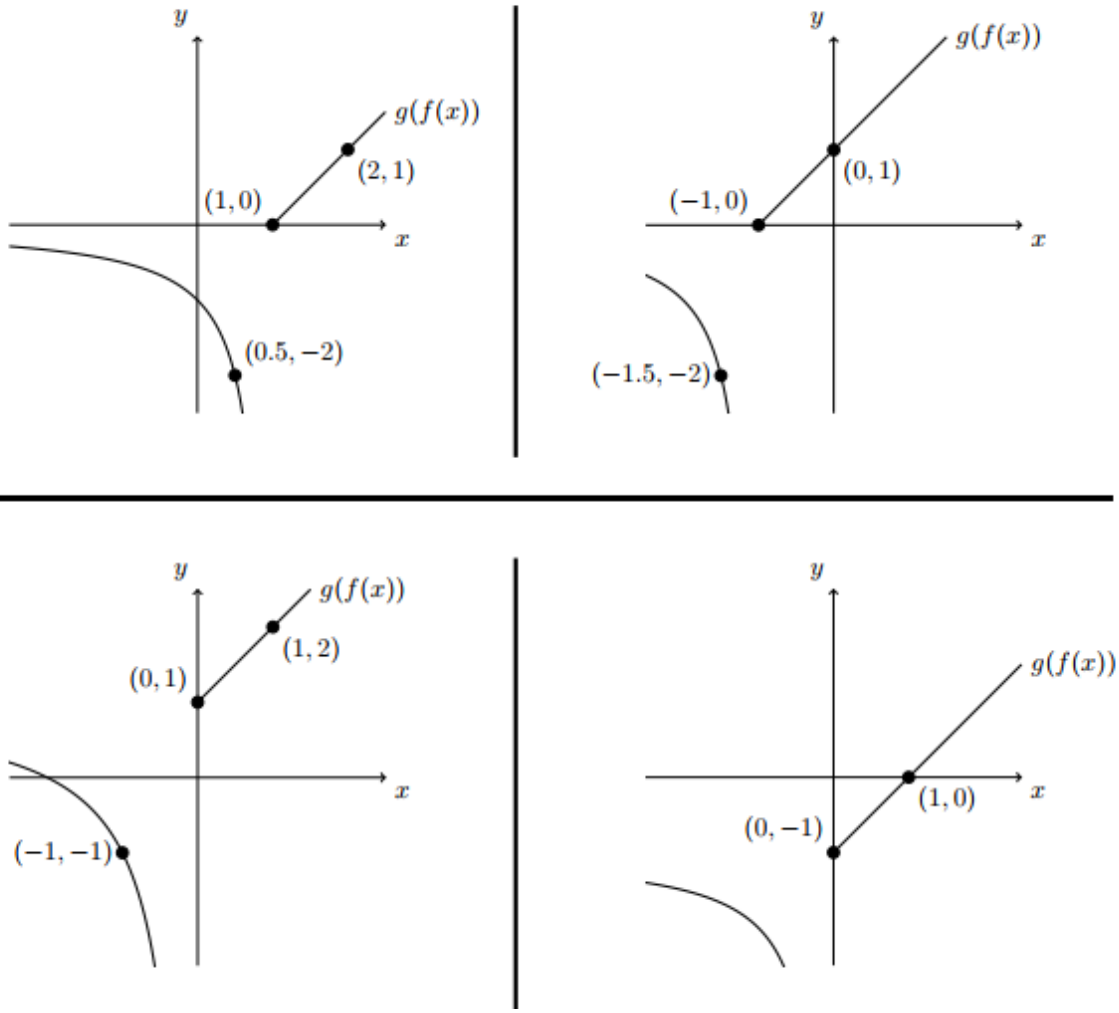








b. [3 points] Which of the following could be the graph of $y = g(f(x))$? **Circle exactly one of the four graphs below.**



10. Let y = g(x) be defined as follows

$$g(x) = \begin{cases} 3 - x \ \ when \ \ x < 2 \\ 2 \ \ \ if \ \ \ x = 2 \\ \dfrac{x}{2} \ \ if \ \ \ x > 2 \end{cases}$$

Sketch the curve.

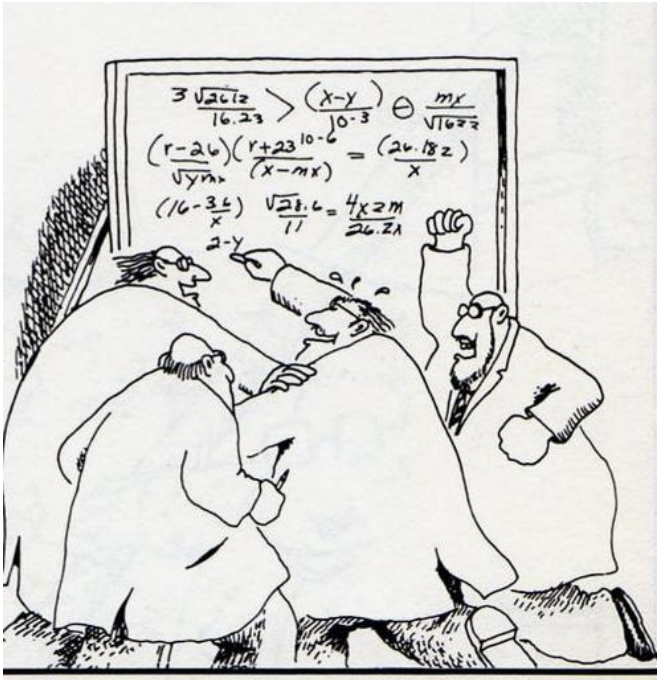
11. Does the limit of g(x) as x→∞ exist?

$$g(x) = \frac{3x^2 - 4x + 1}{x^4 - 1}$$

If so, find it; if not explain!

To be pleased with one's limits is a wretched state.

- **Johann Wolfgang von Goethe** (1749 - 1832)



"Go for it, Sidney! You've got it! You've got it! Good hands! Don't choke!"