The following problems are taken from a *precalculus final exam* at the University of Michigan.

Problem 1

a. [4 points] The graph of a polynomial $p(x)$ is shown below. The following facts are known about $p(x)$:

i) The only zeros of $p(x)$ are $x = -2$ and $x = 3$.

ii) The degree of $p(x)$ is at most four.

iii) The point $(1, 9)$ is on the graph of $p(x)$.
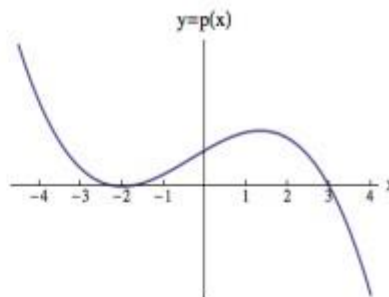
Find a formula for $p(x)$.

$$p(x) = \underline{\hspace{3in}}.$$

b. [5 points] Let

$$R(x) = \frac{(x^2 + 9)(10x + 1)}{7x^3 - x}.$$

Find all the intercepts and all horizontal and vertical asymptotes of the graph $y = R(x)$. If appropriate, write "None" in the answer blank provided. Your answers should be in **exact form**.

i) x-intercept(s): $\underline{\hspace{2in}}$

ii) y-intercept(s): $\underline{\hspace{2in}}$

iii) vertical asymptote(s): $\underline{\hspace{2in}}$

iv) horizontal asymptote(s): $\underline{\hspace{2in}}$

c. [3 points] A law of physics states that the force F (in Newtons) exerted between two objects is inversely proportional to the square of the distance r (in meters) between them, and $F = 30$ when $r = 7$. Find a formula for F in terms of r.

$$F(r) = \underline{\hspace{1.5in}}$$

Problem 2

3. [8 points] The graphs of the functions $f(x)$ and $g(x)$ are shown below. The domain of $f(x)$ and $g(x)$ is $0 \le x \le 7$.
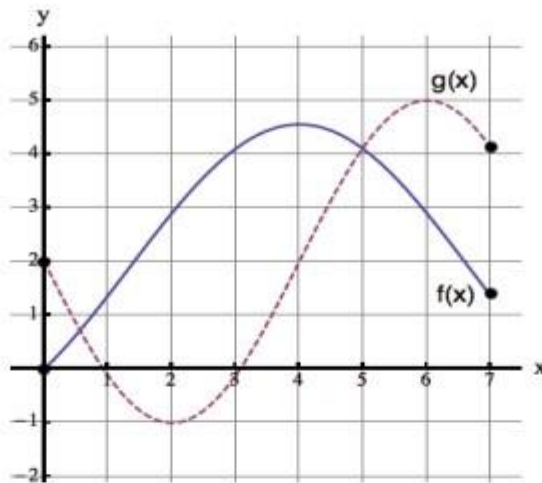
a. [4 points]

i) Find the range of $g(x)$. Use interval notation or inequalities in your answer.

Answer: _____

ii) For which values of $0 \le x \le 7$ is the function $g(x)$ concave down? Use interval notation or inequalities in your answer.

Answer: _____

iii) For which values of $0 \le x \le 7$ is the function $g(x)$ increasing? Use interval notation or inequalities in your answer.

Answer: _____

b. [4 points] Define the functions:

$$D(x) = g(x) - f(x) \qquad \text{and} \qquad R(x) = \frac{g(x)}{f(x)}.$$

i) For which values of $0 \le x \le 7$ is the function $D(x)$ negative? Use interval notation or inequalities in your answer.

Answer: _____

ii) Find the domain of the function $R(x)$. Use interval notation or inequalities in your answer.

Answer: _____

Problem 3

[11 points] A package is thrown from an airplane. The height of the package (in meters) above the ground t seconds after it was thrown from the airplane is given by the function

$$H(t) = -5t^2 - 10t + 160.$$

a. [2 points] What is the height of the airplane at the time in which the package is thrown? Include units.

Height= _____

b. [3 points] How many seconds does it take for the package to be 10 meters above the ground? Find your answer algebraically. Show all your work.

Answer: _____

c. [2 points] What is the range of the function $y = H(t)$ in the context of this problem? Give your answer using either interval notation or inequalities.

Answer: _____

d. [4 points] Another package is released from an airplane at a higher altitude. In this case, the downward velocity $V(t)$ (in meters per second) of the package t seconds after it was released is given by the function

$$V(t) = 50 - 50e^{-0.2t}$$

How long does it take for the package to have a downward velocity of 30 meters per second? Find your answer algebraically. Show all your work step by step. Your answer must be in **exact form**.

$t =$ _____