CLASS DISCUSSION: 12 NOVEMBER 201

VETERAN'S DAY

THE RIEMANN INTEGRAL

Georg Friedrich Bernhard Riemann

(1826 – 1866)

1. Using the *area interpretation* of the Riemann integral, evaluate each of the following:

   (a) $\int_{-2}^{1} |x|\,dx$

   (b) $\int_{-3}^{2} |3x+4|\,dx$

(c) $\int_0^1 \sqrt{1-x^2}\, dx$

(d) $\int_{-3}^{3} x^{1789} \sin(x^2+1)\, dx$

(e) $\int_0^{2\pi} \cos x\, dx$

2. Suppose that $\int_{-2}^{3} \left(f(x)+1 \right) dx = 0.$ Evaluate $\int_{-2}^{3} \left(f(x)-x \right) dx.$

3. Let *g* be a continuous function on the interval [-5, 5]. Suppose that

$$\int_0^5 g(x)\, dx = 4$$

Evaluate each of the following Riemann integrals:

(a) $\int_0^5 \left(g(x)+3 \right) dx$

(b) $\int_{-2}^{3} g(x+2)\, dx$

(c) $\int_{-5}^{5} g(x)\, dx \;\; if \;\; g \, is \; even$

(d) $\int_{-5}^{5} g(x)\, dx \;\; if \;\; g \, is \; odd$

4. Find the constants *a* and *b* that *maximize* the value of the definite integral:

$$\int_a^b \left(4-x^2 \right) dx$$

Justify your answer!

5. By using an appropriate Riemann sum, determine:

$$\lim_{n\to\infty} \frac{1}{n^4} \sum_{j=1}^{n} j^3$$

6. Find a formula for $\int_a^b x\,dx$

7. Express the *average value* of each of the following functions as a Riemann integral. *(Do not try to evaluate.)*

(a) f(x) = sin x over [0, π]

(b) g(x) = $(x - 1)^2$ over [0, 3]

(c) h(x) = (ln x) / x over [1, 4]

(d) s(t) = cosh t over [0, ln 2]

8. State the major properties of the Riemann integral.

9. Suppose that *h* is integrable and that $\int_{-1}^{1} h(x)\,dx = 0 \;\; and \; \int_{-1}^{3} h(x)\,dx = 6.$

Find:

$(a)\;\; \int_1^3 h(x)\,dx$

$(b)\;\; \int_1^3 \left(5h(x) + 3\right)dx$

10. Suppose that *f* and *h* are integrable and that

$$\int_1^9 f(x)\,dx = -1, \;\; \int_7^9 f(x)\,dx = 5 \; and \; \int_7^9 h(x)\,dx = 4$$

Find:

$(a)\;\; \int_1^9 -3f(x)\,dx$

$(b)\;\; \int_7^9 \left(f(x) + h(x)\right)dx$

$(c)\;\; \int_7^9 (5f(x) - 3h(x))\,dx$

$(d)\;\; \int_1^7 (f(x) - |x - 4|)\,dx$

11. Given the formula for $\int_a^b x^2 dx$, find the *average value* of:

(a) $f(x) = x^2 - 1$ over [2, 4]

(b) $g(x) = (x - 2)^2$ over [0, 2]

(c) $h(x) = 5 - 3x - 4x^2$ over [0, 2]

I'm very good at integral and differential calculus,
I know the scientific names of beings animalculous;
In short, in matters vegetable, animal, and mineral,
I am the very model of a modern Major-General.
About binomial theorems I'm teeming with a lot of news,
With many cheerful facts about the square on the hypotenuse.

\- W. S. Gilbert, **The Pirates of Penzance** (1879)