MATH 161 CLASS DISCUSSION

1 OCTOBER

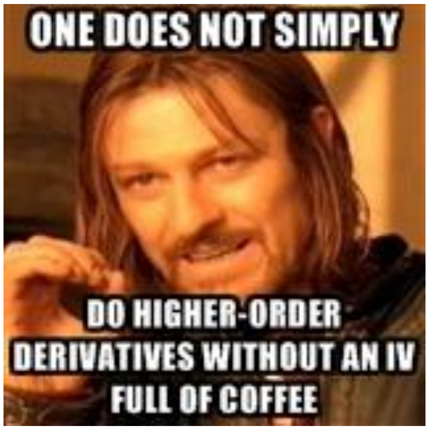
- **The Game of Antiderivatives: A first look at the method of "judicious guessing."**

Find an antiderivative for each of the following functions

(a) $\cos x$ (b) 2018 (c) $6x^5 + 5x^4 + e^5$ (d) $\sec^2 x$ (e) $x^9 + 7x^3 + 1$

(f) $5e^x + \sec x \tan x$ (g) $19 \sin x$ (h) $x - \cos x + 5$

HIGHER-ORDER DERIVATIVES



1. Find the first *three* derivatives of each of the following functions.

$(A)\quad y = ax^2 + bx + c$

$(B)\quad y = 2x^3 + \dfrac{1}{x^2} + e^x$

(Here assume that the shortcut for differentiating x^n is valid for negative values of n.)

$(C)\quad y = xe^x$

$(D)\quad y = \sin x$

$(E)\quad y = x \sin x$

$(F)\quad y = x^{101}$

2. (a) If $(d/dx)e^{4x} = 4e^{4x}$, find $(d^{199}/dx^{199})\, e^{4x}$.

(b) If $(d/dx) \sin 5x = 5 \cos 5x$, and $(d/dx) \cos 5x = - 5 \sin 5x$, find $(d^{2018}/dx^{2018}) \sin 5x$.

3. If $f(x) = x^{1/2}$, find $f^{(4)}(x)$. *(Here assume that the shortcut for differentiating x^n is valid for rational n.)*

4. If $x(t) = 3t^3 - 4t + 1$ is the position (measured in meters) of Charlotte on the x-axis at time t (measured in hours), find Charlotte's *velocity* and *acceleration* at time t = 2 hrs.

5. If $F(x) = x^m$, find $F^{(m)}(x)$. (Assume that m is a positive integer.)

6. Let y = ln x. Given that dy/dx = 1/x, find d^4y/dx^4. Can you find $d^{10}y/dx^{10}$?

7. *(University of Michigan)* Consider the following table giving values, rounded to three decimal places, of a function $f(x)$.

x	0	0.5	1
$f(x)$	0	0.247	0.841

(a) Estimate $f'(1)$. Be sure it is clear how you obtain your answer.

(b) Estimate $f(1.25)$ being sure your work is clear.

(c) Estimate $f''(1)$. Again, be sure that it is clear how you obtain your answer.

(d) Based on your work in (a) and (c), is your estimate in (b) an over- or underestimate? Explain.

8. *(University of Michigan)* A paperback book (definitely not a valuable calculus textbook, of course) is dropped from the top of Mertz hall (which is 40 m high) towards a very large, upward pointing fan. The average velocity of the book between time $t = 0$ and later times is shown in the table of data below (in which t is in seconds and the velocities are in m/s).

between t = 0 seconds and t =	**1**	**2**	**3**	**4**	**5**
the average velocity is	**−5**	**−10**	**−11.67**	**−9**	**−7.2**

a) Fill in the following table of values for the height $h(t)$ of the book (measured in meters). Show how you obtain your values.

t	0	1	2	3	4	5
$h(t)$	40	______	______	______	______	______

b) Based on your work from (a), is $h''(1) > 0$, < 0, or $= 0$? Is $h''(3) > 0$, < 0, or $= 0$? Explain.

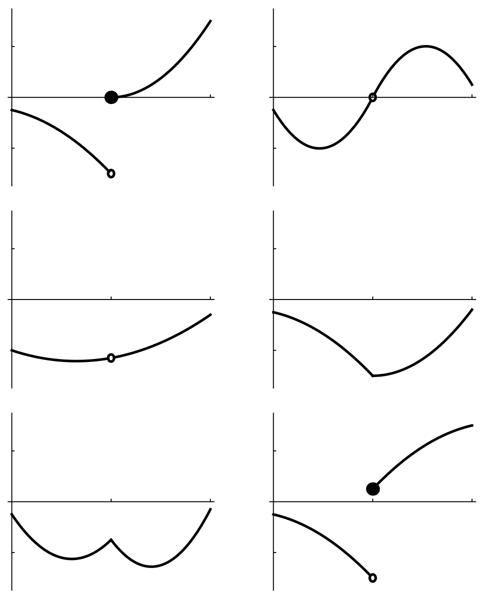
9. For each of the descriptions of a function f that follow, indicate which of the graphs match the description. For each description there may be no, one, or several graphs that match; write **none** if no graphs match the description. You may need to use a graph more than once. In each case, you should assume that f is defined only on the domain [0, 2].

a) $f''(x) < 0$ for $x < 1$ and $f''(x) > 0$ for $x > 1$; $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$; and $f(x)$ is continuous everywhere except at $x = 1$.

b) $f''(x) > 0$ for all $x < 1$; $f''(x) < 0$ for all $x > 1$; and $f(x)$ is differentiable everywhere except at $x = 1$.

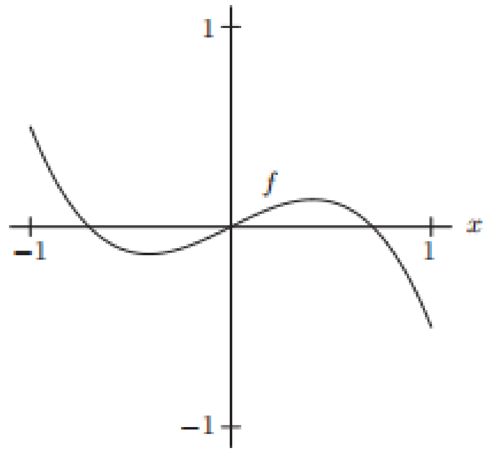
c) $f''(x) < 0$ for all $x < 1$; $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$; and $f(x) < 0$ for all $x = 1$.

d) $f''(x) < 0$ for $x < 1$ and $f''(x) > 0$ for $x > 1$; $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$; and $f(x)$ is differentiable everywhere except at $x = 1$.



10. The graph of a function f is given in the Figure below. If f is a polynomial of degree 3, then the value of $f'''(0)$ *is*

(a) Positive (b) Negative (c) Zero



11.

The graph of a function f is given in Figure 3.5. If f is a polynomial of degree 3, then the values of $f'(0)$, $f''(0)$, and $f'''(0)$ are (respectively)

(a) $-,+,+$ (b) $-,-,-$ (c) $-,+,-$
(d) $-,+,+$ (e) $+,-,+$ (f) $+,+,+$

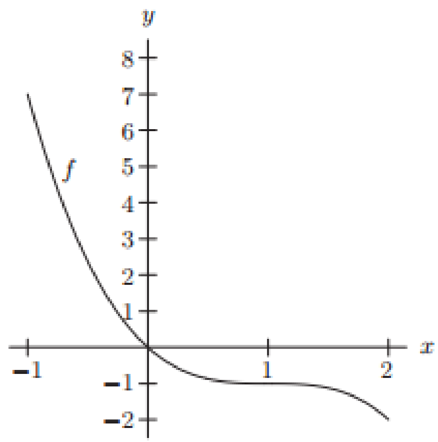
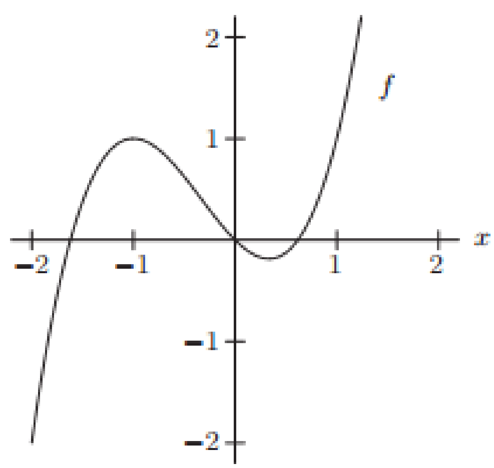
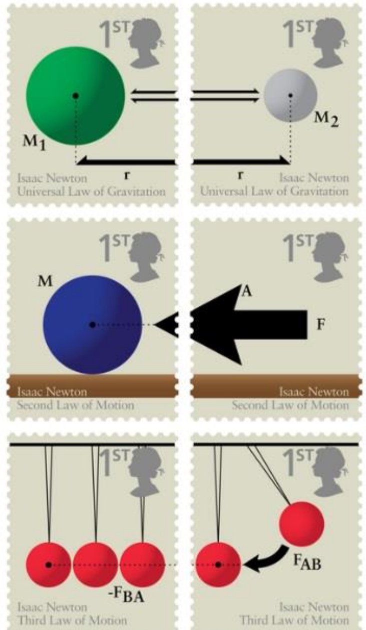


Figure 3.5

12. The graph of a function f is given below. If f is a polynomial of degree 3, then the values of $f'(0), f''(0), and\ f'''(0) are\ (respectively)$

(a) $-,-,+$ (b) $-,0,-$ (c) $-,+,-$
(d) $-,+,+$ (e) $+,-,+$ (f) $+,+,+$




1ST
M1
M2
r
Isaac Newton
Universal Law of Gravitation
M
A
F
Isaac Newton
Second Law of Motion
-FBA
FAB
Isaac Newton
Third Law of Motion