DISCUSSION QUESTIONS: 16 OCTOBER 2019

LOGARITHMIC DIFFERENTIATION

"First you forget logarithms. Then you forget how to do long division. Then the multiplication table begins to go . . ."

1. (a) Can you find a formula for d/dx (f (x) g(x) h(x))? (Called *Leibniz rule*.)

(b) Can you extend this result to a product rule for four or more factors?

(c) Using your result from (b), compute d/dx $\{5(x^3)$ (cos x) (ln x) $e^x\}$

(d) Find any and all critical points of the function: $y = (x^2 + 3)(x - 5)\, e^x$

2. *(UC Davis)* *Logarithmic differentiation* is a means of differentiating algebraically complicated functions or functions for which the ordinary rules of differentiation do not apply. For example, if you wish to differentiate expressions where a variable is raised to a variable power, logarithmic differentiation is an invaluable technique.

(a) Differentiate $g(x) = x^{2x+3}$.

(b)  An example of two **COMMON INCORRECT SOLUTIONS** are:

1.) $D\{x^{(2x+3)}\} = (2x+3)x^{(2x+3)-1} = (2x+3)x^{(2x+2)}$

and

2.) $D\{x^{(2x+3)}\} = x^{(2x+3)}(2)\ln x$.

BOTH OF THESE SOLUTIONS ARE WRONG because the ordinary rules of differentiation do not apply. Logarithmic differentiation will provide a way to differentiate a function of this type. It requires deft algebra skills and careful use of the following unpopular, but well-known, properties of logarithms. Though the following properties and methods are true for a logarithm of any base, only the natural logarithm, ln x, will be used in this problem set.

3.

PROPERTIES OF THE NATURAL LOGARITHM

1. $\ln 1 = 0$.
2. $\ln e = 1$.
3. $\ln e^x = x$.
4. $\ln y^x = x \ln y$.
5. $\ln(xy) = \ln x + \ln y$.
6. $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$.

4. **Avoid the following FALSE FRIENDS:**

1. $\ln(x+y) = \ln x + \ln y$.
2. $\ln(x-y) = \ln x - \ln y$.
3. $\ln(xy) = \ln x \ \ln y$.
4. $\ln\left(\frac{x}{y}\right) = \frac{\ln x}{\ln y}$.
5. $\frac{\ln x}{\ln y} = \ln x - \ln y$.

5. *(UC Davis)* The following problems range in difficulty from average to challenging.

- *PROBLEM 1:* Differentiate $y = x^x$.
- *PROBLEM 2:* Differentiate $y = x^{(ex)}$.
- *PROBLEM 3 :* Differentiate $y = (3x^2+5)^{1/x}$
- *PROBLEM 4 :* Differentiate
  $y = (\sin x)^{x^3}$
- *PROBLEM 5 :* Differentiate
  $y = 7x(\cos x)^{x/2}$
- *PROBLEM 6 :* Differentiate
  $y = \sqrt{x}^{\sqrt{x}} e^{x^2}$
- *PROBLEM 7 :* Differentiate
  $y = x^{\ln x}(\sec x)^{3x}$
- *PROBLEM 8 :* Differentiate
  $y = \frac{(\ln x)^x}{2^{3x+1}}$
- *PROBLEM 9 :* Differentiate
  $y = \frac{x^{2x}(x-1)^3}{(3+5x)^4}$

.

- *PROBLEM 10:* Consider the function

$$f(x) = \frac{x^5 e^x (4x+3)}{5^{\ln x}(3-x)^2}$$

.

Find an equation of the line tangent to the graph of *f* at *x* = 1.

- *PROBLEM 11:* Consider the function

$$f(x) = \pi^2 + 2^x + x^2 + x^{1/x}$$

Determine the slope of the line perpendicular to the graph of *f* at *x* = 1.

- *PROBLEM 12:* Differentiate $f(x) = x^{\left(x^{(x^4)}\right)}$. Then determine the slope of the normal line to the graph of *f* at *x*=1.

Additional exercises:

6. Find $\frac{dy}{dx}$ *if* $x^y = y^x$. Hint: Use logarithmic differentiation.

7. Using *logarithmic differentiation*, find dy/dx if:

(a) $y = \dfrac{x(x-9)^5\sqrt{x+5}}{x^5+99}$

(b) $y = 7(x-9)^3(x^3+x+1)^5$

(c) $y = (\sin^3 x)(\tan^5 x)(\ln x)^2$

(d) $y^x = (x+1)^{3x}$

(e) $y^{\sin x} = (\ln x)^y$

(f) $f(x) = (5-3x^2)^7\sqrt{6x^2+8x-2019}$