## RELATED RATES, CONTINUED

**(University of Michigan problems)**

1.   Walking through the Loyola campus one autumn day, Albertine sees a squirrel running down the trunk of a tree. The trunk of the tree is perfectly straight and makes a right angle with the ground. She stops 20 feet away from the tree and lies down on the ground to watch the squirrel.

Suppose h(t) is the distance in feet between the squirrel and the ground, and θ(t) is the angle in radians between the ground and Albertine's line of sight to the squirrel, with *t* being the amount of time in seconds since she stopped to watch the squirrel.
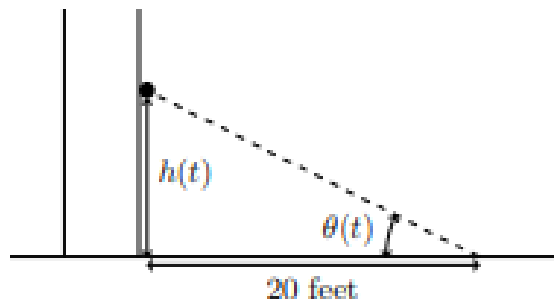
(a) Write an equation relating h(t) and θ(t). (Hint: Use the tangent function.)

(b) If θ(t) is decreasing at 1/5 of a radian per second when θ(t) = π/3, how fast is the squirrel moving at that time?

(c) For the last second, before the squirrel reaches the ground, it is moving at a constant speed of 20 feet per second. Suppose θ′ (t) = −3/4 at some point during this last second. How high is the squirrel at this time?

2.   Suppose that a ring made entirely of gold and platinum is made from *g* ounces of gold and *p* ounces of platinum and that gold costs *h* dollars per ounce and platinum costs *k* dollars per ounce. Then the *value*, in dollars, of the ring is given by **v = gh + pk**.

(a) Swann has a ring made entirely of gold and platinum. Swann's ring is made from 0.25 ounces of gold and 0.15 ounces of platinum. Suppose that the cost of gold is *decreasing* at an instantaneous rate of $20 per ounce per year while the cost of platinum is *increasing* at an instantaneous rate of $30 per ounce per year. At what instantaneous rate is the value of Swann's ring increasing or decreasing? Remember to include units in your answer.

(b) Odette wants to design a ring made entirely of gold and platinum with a current value of $900. Currently, gold costs $1200 per ounce and platinum costs $1500 per ounce. Let w(p) be the total weight of Odette's ring, in ounces, if p ounces of platinum are used.

   (i) In the context of this problem, what is the domain of w(p)?
   (ii) Find a formula for w(p). No variables other than p should appear in your answer.
   (iii) How much gold and platinum should be in the ring if Odette wants to *minimize* the weight of the ring?

3. A wicked villain decides to relax with handmade chocolate before he heads to his farmhouse. The process of making the chocolate involves pouring molten chocolate into a mold. The mold is a cone with height 60 mm and base radius 20 mm. Freddy places the mold on the ground and begins pouring the chocolate through the apex of the cone. A diagram of the situation is shown below.
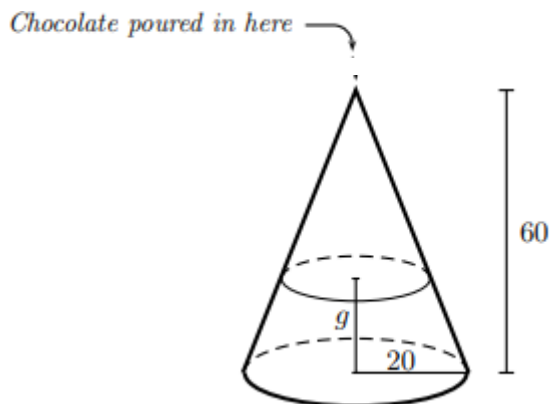
In case they are helpful, recall the following formulas for a cone of radius r and height h:
$$\text{Volume} = \tfrac{1}{3}\pi r^2 h \quad \text{and} \quad \text{Surface Area} = \pi r(r + \sqrt{h^2 + r^2}).$$

   (a) Let g be the depth of the chocolate, in mm, as shown in the diagram above. What is the value of g when Freddy has poured 20,000 mm^3 of chocolate into the mold? Show your work carefully, and make sure your answer is accurate to at least two decimal places.

   (b) How fast is the depth of the chocolate in the mold (g in the diagram above) changing when Freddy has already poured 20,000 mm^3 of chocolate into the mold if he is pouring at a rate of 5,000 mm^3 per second at this time? Show your work carefully and make sure your answer is accurate to at least two decimal places. Be sure to include units.
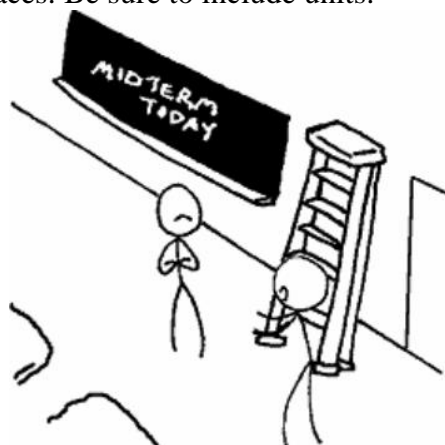
SURREAL TEST-TAKING:
ON A CALCULUS TEST OVER
RELATED-RATE PROBLEMS,
INSIST ON DETERMINING YOUR
ANSWERS EMPIRICALLY THROUGH
EXPERIMENTATION AND OBSERVATION.

WHEN YOUR PROFESSOR OBJECTS,
REFUSE TO GIVE UP THE LADDER
AND STOPWATCH.