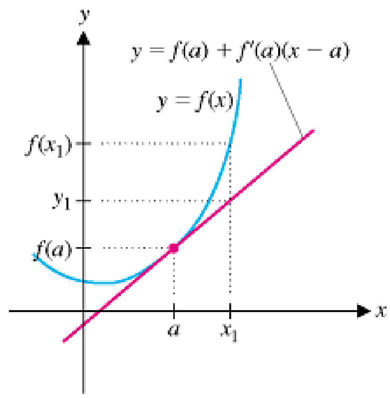
CLASS DISCUSSION: 3 OCTOBER

LINEAR APPROXIMATIONS

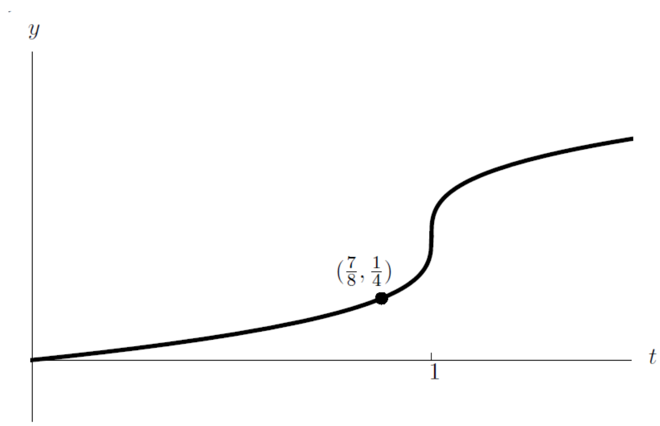


1. Find the linearization of the function $f(x) = \sqrt{x+3}$ at the point x = 1 and use it to approximate $\sqrt{3.98}$ *and* $\sqrt{4.05}$. For each approximation, is it an underestimate or an overestimate? Explain. (Here you may use the power rule short cut.)

2. Find the linearization of the function f(x) = sin x at the point x = π/6.

3. Find the linearization of the function $f(x) = (1+x)^{-3}$ at the point x = 0 and use it to approximate the value of $\frac{1}{1.003^3}$. *Is your* approximation an underestimate or an overestimate? Explain.

4. *(U. Michigan)* Given below is the graph of a function h(t). Suppose j(t) is the local linearization of h(t) at t = 7/8.



(a) Given that $h'\left(\frac{7}{8}\right) = \frac{2}{3}$, *find an expression* for *j*(t).

(b) Use your answer from (a) to approximate h(1).

(c) Is the approximation from (b) an over- or under-estimate? Explain.

(d) Using j(t) to estimate values of h(t), will the estimate be more accurate at t=1 or t = ¾? Explain.

5. [12 points] Preparing a pot of soup for dinner, Billy heats the soup to boiling and then removes it from the stove. The function $H(t)$ gives the temperature of the soup in °F as a function of the number of minutes since it was removed from the stove. Assume that $H(0) = 212$ and $H'(0) = -2.7$, and that Billy's kitchen is carefully air-conditioned to remain at a comfortable 68 °F at all times. Throughout this problem, be sure to **include units** in your answers, where applicable.

a. [3 points] Approximate $H(1.5)$.

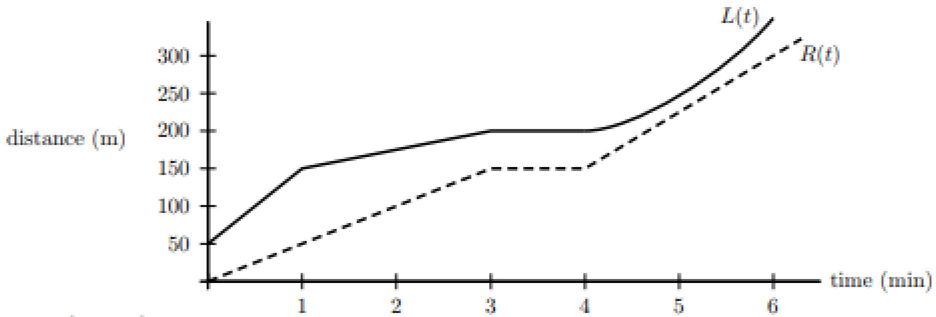
b. [3 points] Five minutes after removing the soup from the stove, Billy remarks to himself: "In the next 30 seconds, I expect the soup to cool by about 0.875 °F." Since he is both an excellent chef and a student of Math 115, his statement is consistent with the actual value of the derivative of the function H. Based on this information, find $H'(5)$, and justify your answer.

c. [3 points] Assume that the concavity of $H(t)$ is the same on its entire domain. Based on your answer to part (b) and the given information, do you expect that the function $H(t)$ is concave up or concave down? Briefly explain your answer.

d. [3 points] Called off on important business, Billy leaves the pot of soup uneaten. Approximate $H'(300)$. (You may use the practical interpretation of $H(t)$, but be sure to explain your answer.)

Problem 6

5. [14 points] Elphaba the squirrel is panicking because she has noticed that a human, Erin, is watching her. Elphaba starts to run and Erin is soon in full-blown pursuit as they both run straight down the street. Let $R(t)$ be Erin's distance from their starting point (in meters) t minutes after the chase begins and $L(t)$ be Elphaba's distance from the starting point (in meters) t minutes after the chase begins. The graphs of $R(t)$ (dashed) and $L(t)$ (solid) for the first 6 minutes of the chase are shown below.



a. [1 point] Which of the following expressions gives the distance, in meters, between Elphaba and Erin t minutes after the chase begins? *Circle the* ONE *best option.*

i. $L'(t) - R'(t)$ ii. $R'(t) - L'(t)$ iii. $L(t) - R(t)$ iv. $R(t) - L(t)$ v. $R^{-1}(L(t))$ vi. $L^{-1}(R(t))$

b. [2 points] What is Erin's velocity when $t = 0.5$? *Be sure to include units.*

Answer: ______________________

c. [3 points] During which of the following time periods is Erin gaining on Elphaba? *Circle* ALL *correct answers.*

i. $0 \leq t \leq 0.75$ ii. $1.25 \leq t \leq 2.75$ iii. $3.25 \leq t \leq 3.75$ iv. $4.25 \leq t \leq 4.75$ v. $5.25 \leq t \leq 6$

d. [3 points] During which of the following time periods is there at least one time when Erin and Elphaba are travelling at the same speed? *Circle* ALL *correct answers.*

i. $0.25 \leq t \leq 0.75$ ii. $1.75 \leq t \leq 2.25$ iii. $2.25 \leq t \leq 2.75$ iv. $3.25 \leq t \leq 3.75$ v. $4.75 \leq t \leq 5.25$

e. [2 points] Circle all of the following events that could be occurring between the 3rd and the 4th minutes.

i. Elphaba is getting further from Erin.

ii. Erin is tying her shoe.

iii. Elphaba has stopped.

iv. Erin is gaining on Elphaba.

f. [3 points] What is Elphaba's average velocity over the first 3 minutes of the chase? *Be sure to include units.*

7.

(U. Michigan)

[12 points] In Srebmun Foyoj, Maddy and Cal are eating lava cake. Let $T(v)$ be the time (in seconds) it takes Maddy to eat a v cm^3 serving of lava cake. Assume $T(v)$ is invertible and differentiable for $0 < v < 1000$. Several values of $T(v)$ and its first and second derivatives are given in the table below.

v	10	15	60	100	150	200	300
$T(v)$	11	22	84	194	393	513	912
$T'(v)$	2.4	1.9	1.8	3.6	3.7	0.9	17.5
$T''(v)$	-0.11	-0.08	0.05	0.04	-0.04	-0.05	0.59

Remember to show your work carefully.

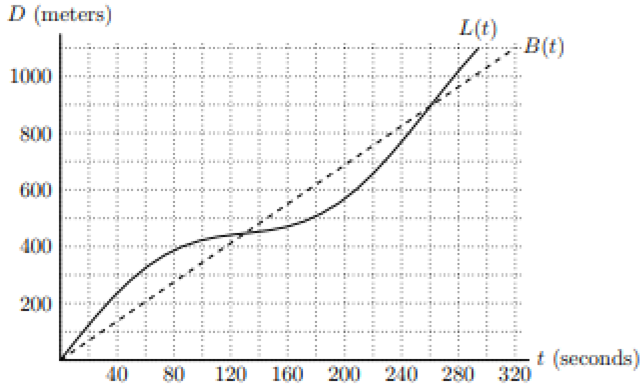
a. [4 points] Use an appropriate linear approximation to estimate the amount of time it takes Maddy to eat a 64 cm^3 serving of lava cake. *Include units.*

b. [4 points] Use the quadratic approximation of $T(v)$ at $v = 200$ to estimate $T(205)$. (Recall that a formula for the quadratic approximation $Q(x)$ of a function $f(x)$ at $x = a$ is $Q(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2$.)

c. [4 points] Let $C(v)$ be the time (in seconds) it takes Cal to eat a v cm^3 serving of lava cake, and suppose $C(v) = T(\sqrt{v})$. Let $L(v)$ be the local linearization of $C(v)$ at $v = 100$. Find a formula for $L(v)$. Your answer should not include the function names T or C.

Problem 8

[11 points] Link and Boots decided to have a race down a straight portion of Pauline Boulevard that is 1.1 kilometers long. Let $L(t)$ and $B(t)$ be Link's and Boots's respective distances from their starting point t seconds after the race began. A graph of $L(t)$ and $B(t)$ is shown below.



a. [1 point] Who won the race? (Circle your answer.)

Link Boots

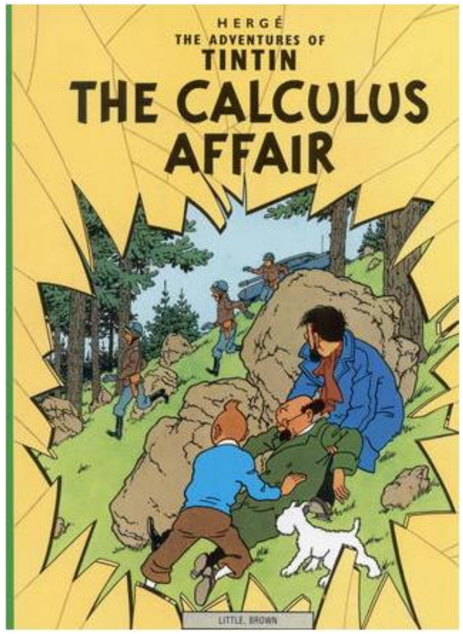
b. [2 points] Estimate the times at which Link and Boots were running at the same speed.

c. [3 points] Estimate Link's average velocity over the first 100 seconds of the race. Include units.

d. [3 points] Estimate Link's instantaneous velocity 40 seconds after the race began. Include units.

e. [2 points] 160 seconds after the race began, is Link's acceleration positive, negative, or equal to zero? (Circle your answer.)

positive negative zero



The Game of Anti-Derivatives:

Find an anti-derivative of each of the following functions:

(a) x^{99} (b) $\frac{3}{x^2}$ (c) $\frac{1+x}{\sqrt{x}}$ (d) $x(x+7)$ (e) $(x^3 + x + 1)(x^3 - 2x)$

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