MATH 161 DISCUSSION: 12 SEPTEMBER 2018

Part I: trigonometric limits; one-sided lmits

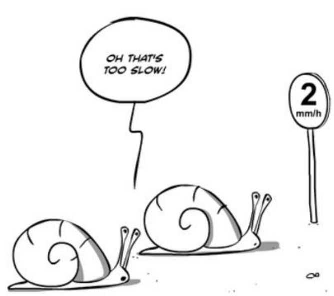
A Evaluate each of the following limits or explain why the limit fails to exist.

1. $\lim\limits_{x\to 0}\dfrac{\sin 4x}{x}$
2. $\lim\limits_{x\to 0}\dfrac{\tan 5x}{x}$
3. $\lim\limits_{x\to 0}\dfrac{\sin 2x}{\sin 8x}$
4. $\lim\limits_{x\to \infty}\dfrac{\sin 13x}{x}$
5. $\lim\limits_{x\to 0}\dfrac{\cos 3x}{x}$
6. $\lim\limits_{x\to 0+} x\sin\left(\dfrac{1}{x}\right)$
7. $\lim\limits_{x\to 0}\dfrac{\cos 11x}{\cos 13x}$
8. $\lim\limits_{x\to 0}\dfrac{\tan^2 x}{x^2}$
9. $\lim\limits_{x\to 0}\dfrac{\sin^2 x}{x}$
10. $\lim\limits_{x\to 0+}\dfrac{|x|}{x}$
11. $\lim\limits_{x\to 5-}\dfrac{x(x-5)(x-3)^2}{|x-5|}$
12. $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x}$
13. $\lim\limits_{x\to 5}\sqrt{\dfrac{x-5}{x+1}}$
14. $\lim\limits_{x\to 0}\dfrac{\sin(\sin x)}{\sin x}$
15. $\lim\limits_{x\to 0} x\csc x$
16. $\lim\limits_{x\to 3-}\dfrac{(x+4)(x-3)}{|x-3|}$
17. $\lim\limits_{x\to 0}\cos(1/x)$
18. $\lim\limits_{x\to 3-}\sqrt{9-x^2}$
19. $\lim\limits_{n\to 1+}\dfrac{x-1}{|x-1|}$
20. $\lim\limits_{n\to 1-}\dfrac{x-1}{|x-1|}$

B 1. Prove that $\lim\limits_{x\to 0}\dfrac{\sin x}{x}$.

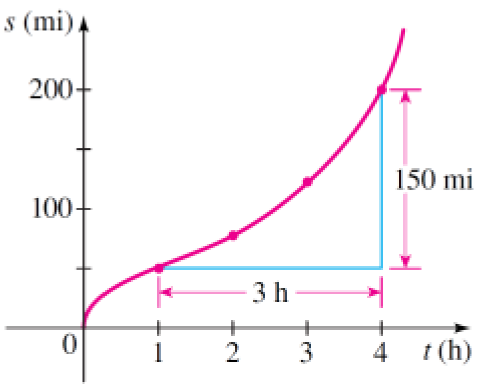
2. Prove, using (1) and a trigonometric identity, that $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x} = 0$.

PART II: average rate of change



1. Suppose that Albertine travels from Hell, Michigan, to Paradise, Michigan, on the Upper Peninsula. Along the way, every few minutes, she records the distance that she has traveled in miles. Suppose

that she begins her trip at time t = 0 (corresponding to noon). Let s(t) be the distance she has traveled at time t. The following is Albertine's graph of position (miles) vs time (hours).



(a) Find Albertine's average speed between 1 pm and 4 pm.

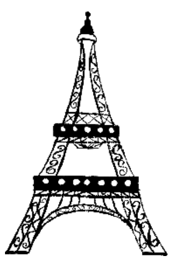
(b) Find her average speed between 2 pm and 3 pm.

Note: Be certain to include appropriate units in your answers!

2. Find the *average rate of change* of the function $f(x) = x^2 + 1$ over the interval

(a) [0, 2] (b) [2, 3] (c) [-2, 0] (d) [1, 1 + h] where h > 0.

(e) Give a *geometric interpretation* for each of these calculations.



3. If Swann drops a bowling ball from the top of the Eiffel Tower, then the distance it will fall (in feet) after *t* seconds is given by the function
$d(t) = 16\,t^2$.
Find the *average* speed of the ball between

(a) t = 1 and t = 4 seconds.

(b) t = a and t = a + h seconds, where a and h are positive.

4. Let g(x) = 3x – 19. Find the *average rate of change* of this function from:

(a) x = 0 and x = 1

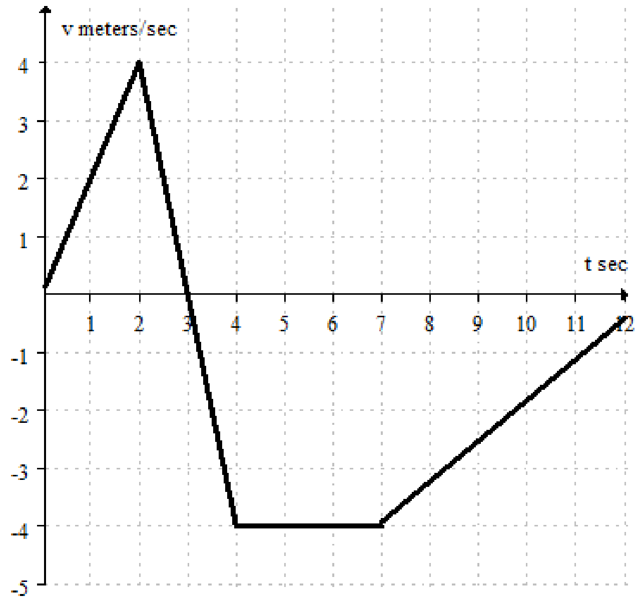
(b) x = 1 and x = 4

(c) x = a and x = a + h

5. Let f(x) = mx + b. Find the *average rate of change of f* over the interval
x = a and x = a + h.

6.
If $x(V) = V^{1/3}$ is the length of the side of a cube in terms of its volume, V, calculate the average rate of change of x with respect to V over the interval $3 < V < 4$ to 2 decimal places.

7. The graph below shows the velocity of a lady bug traveling along a straight line on the classroom floor.

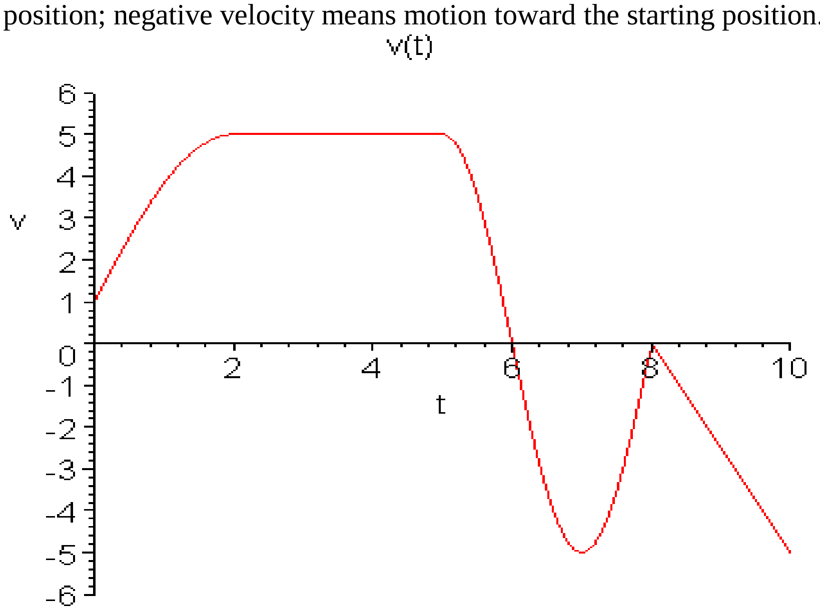


When is the bug moving at a constant speed?
A) Between 4 and 7 seconds.
B) Whenever the velocity is linear with a positive slope.
C) Whenever the velocity is linear with a negative slope.
D) When the velocity is equal to zero.

8. Let $L(R)$ be the amount of board-feet of lumber produced from a tree of radius r (measured in inches). What does $L(16)$ mean in practical terms?

A) The amount of board-feet of lumber produced from a tree with a radius of 16 inches.
B) The radius of a tree that will produce 16 board-feet of lumber.
C) The rate of change of the amount of lumber with respect to radius when the radius is 16 inches (in board-feet per inch).
D) The rate of change of the radius with respect to the amount of lumber produced when the amount is 16 board-feet (in inches per board-foot).

9. Below is the graph of a velocity function. The units on the vertical axis are in *kilometers per hour* and the units on the horizontal axis are in *hours*. Positive velocity means motion away from the starting position; negative velocity means motion toward the starting position.



(a) Create a (brief) story about motion of a person or thing whose velocity is represented by this graph. Your story must include all of the "important" information included in the graph above, including position and times t = 0 and t = 10.)

(b) Sketch a possible graph of the position function of the person or thing in your story on the time interval t = 0 to t = 10.

10. Suppose that C(T) is the cost of heating Albertine's house, in dollars per day, when the *outside* temperature is *T* degrees Fahrenheit. What does C(19) = 8.67 mean in practical terms? (Use appropriate units.)

11. The cost *C* (in thousands of dollars) of building a house that is *x* square feet is given by the function C = F(x). Explain the *meaning* of the statement: F(1600) = 140.

The more you know, the less sure you are.

- Voltaire

I used to love mathematics for its own sake, and I still do, because it allows for no hypocrisy and no vagueness…

- Stendhal, **The Life of Henri Brulard**