DISCUSSION QUESTIONS: 24 SEPTEMBER

SHORTCUTS APPLIED TO CURVE SKETCHING, CONTINUED

I Using the shortcuts of differentiation *when appropriate*, compute the derivative of each of the following functions.

(A) $y = 2018 + 3x - \pi x^4 + e^{1789}$

(B) $y = x \sin x$

(C) $y = \dfrac{x+3}{x+7}$

(D) $y = \dfrac{x}{\sin x}$

(E) $y = x^3 \cos x - x + 1$

(F) $y = (x^2 + 4x - 1)(x^3 + 5x^4 - x^3 + x^2 + 3x + 13)$

(G) $y = \sin^2 x$

II (a) Find the equations of the *tangent* and *normal lines* to the curve

$$y = \frac{x-4}{x+1} \quad at\ x = 3.$$

(b) Find the equations of the *tangent* and *normal* lines to the curve

$y = \sin x$ at $x = \pi/4$.

III Using appropriate shortcuts, find formulas for the derivatives of

$y = \tan x$ and $y = \sec x$.

V Sketch the curve $y = x^3(x - 2)^2$. Over which interval(s) is the graph *rising? falling?* Locate any local maxima or minima.

VI Sketch the curve $y = \frac{x-4}{x+1}$ (cf. problem II a). Over which interval(s) is the graph *rising*? *falling*? Locate any *local maxima* or *minima*.

VII Sketch the curve $y = x e^x$. Over which interval(s) is the graph rising? *falling?* Locate any local maxima or minima.

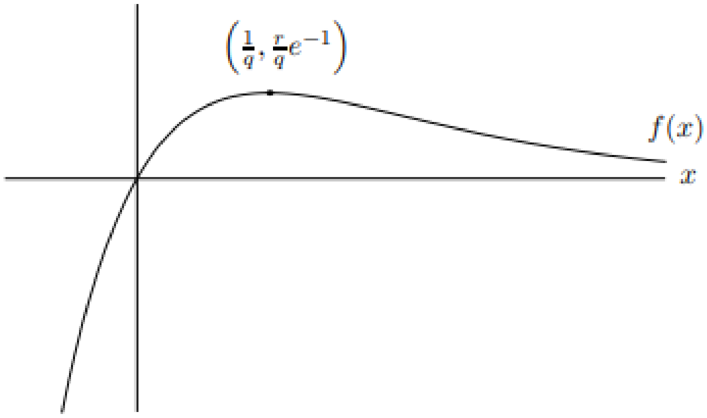
X Sketch the curve $y = 1/x + x^2$ over the interval $(0, \infty)$. Over which interval(s) is the graph *rising*? *falling*? Locate any local maxima or minima.

XI Below is the graph of the function

$$f(x) = rxe^{-qx},$$

where r and q are constants. Assume that both r and q are greater than 1. The function $f(x)$

passes through the origin and has a local maximum at the point $P = \left(\frac{1}{q}, \frac{r}{q}e^{-1}\right)$, as shown in the graph.



a. Justify, using the first-derivative test that the point P is a local maximum.

b. What are the x-coordinates of the global maximum and minimum of $f(x)$ on the domain [0, 1]? (If $f(x)$ does not have a global maximum on this domain, say "no")

c. What are the x-coordinates of the global maximum and minimum of $f(x)$ on the domain $(-\infty, \infty)$? (If $f(x)$ does not have a global maximum on this domain, say "no global maximum", and ~~similarly if $f(x)$~~ does not have a global minimum.)

d. Suppose that $g(x)$ is a function with $g'(x) = f(x)$. Find x-values of all local maxima and minima of $g(x)$. Justify that each maximum you find is a maximum and each minimum is a minimum.