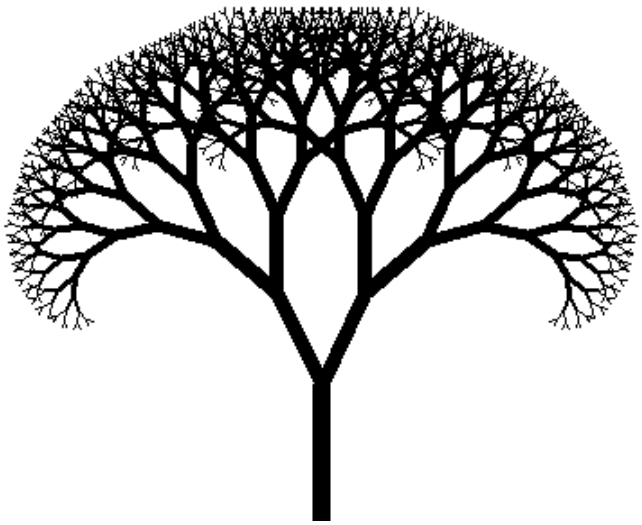
# DISCUSSION: 5 SEPTEMBER

## limits & continuity



***Fractal tree***

**I** Evaluate each of the following limits or explain why the limit fails to exist.

1. $\lim_{x\to 3} \dfrac{x-3}{x^2-5x+6}$

2. $\lim_{x\to 3} \dfrac{x^3-27}{x-3}$

3. $\lim_{x\to 1} \dfrac{x^4-1}{x^2-1}$

4. $\lim_{x\to 1} \dfrac{x+9}{x^2-4}$

5. $\lim_{x\to 1} \dfrac{x^4-1}{x^3-1}$

6. $\lim_{x\to 0} \dfrac{|x|}{x}$

7. $\lim_{x\to 16} \dfrac{\sqrt{x}-4}{x-16}$

8. $\lim_{x\to 1} \dfrac{\frac{1}{x}-1}{x-1}$

9. $\lim_{x\to 4} \sqrt{\dfrac{x+5}{x+12}}$

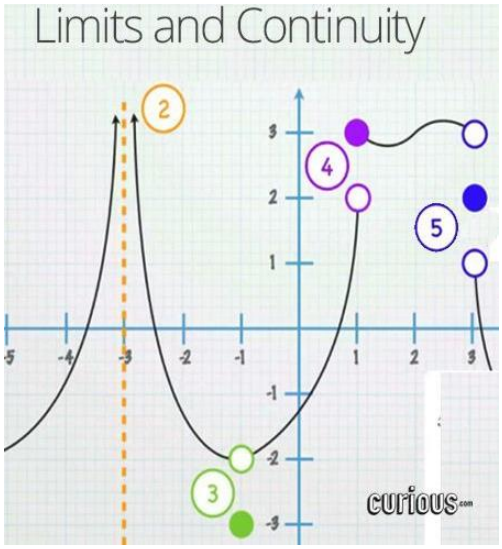
10. $\lim_{x\to 1} \dfrac{x^2-1}{(x-1)^3}$

**II** State the *limit laws*.

**III** Define *continuity* of a function y = f(x) at x = a. What does it mean for a function to be *continuous*?

**IV** (a) For each of the four types of discontinuity (*removable, infinite, jump, essential*) give several examples.

(b) For the graph below, characterize each of the four discontinuities.




(c) Give an example of an *essential discontinuity*.

**V** Consider each of the following functions and the given point on the x-axis. Does the function have a *continuous extension* at the given point? Explain.

1. $f(x)=\dfrac{x-2}{x-3},\ x=3$

2. $G(x)=\dfrac{x^2-9}{x-3},\ x=3$

3. $H(x)=\dfrac{2x^2-13x+20}{3x^2-13x+4},\ x=4$

4. $g(x)=\dfrac{2x^2-13x+20}{3x^2-13x+4},\ x=1/3$

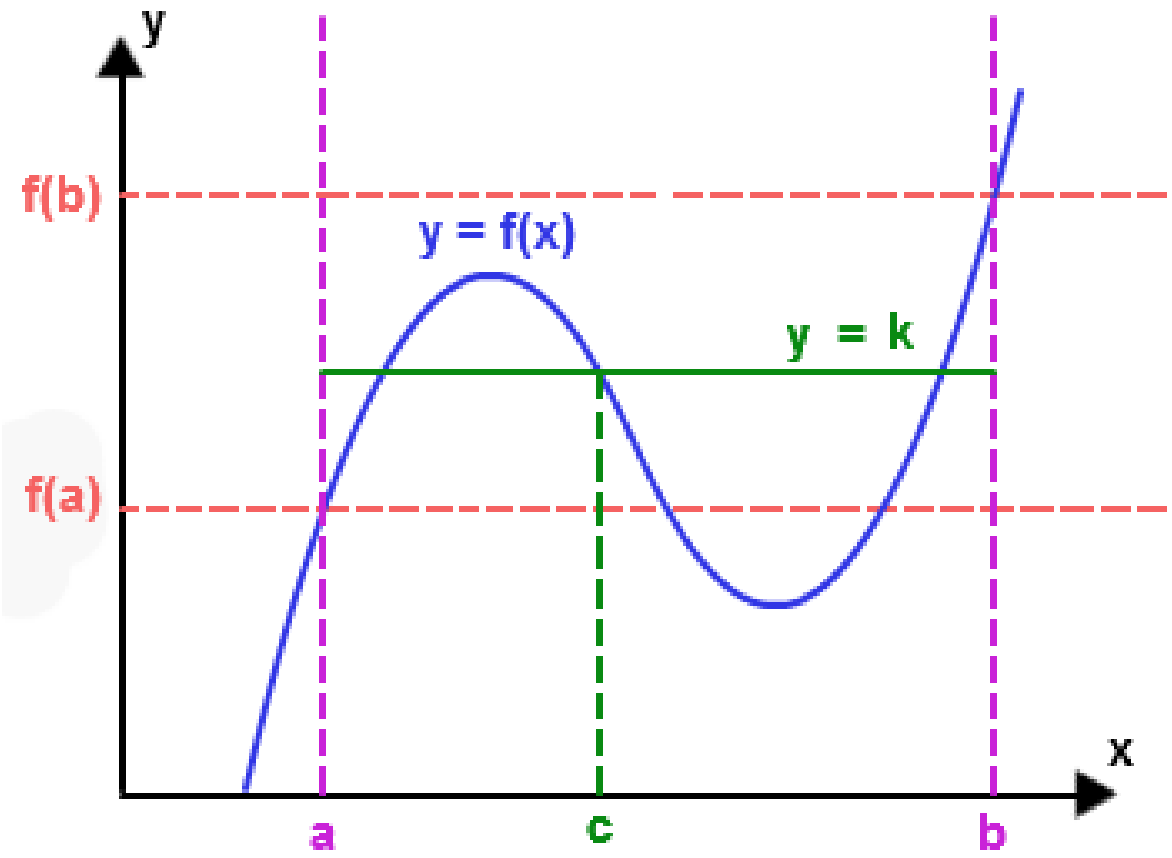
**VI** For which value of *a* is the following function *continuous everywhere?*

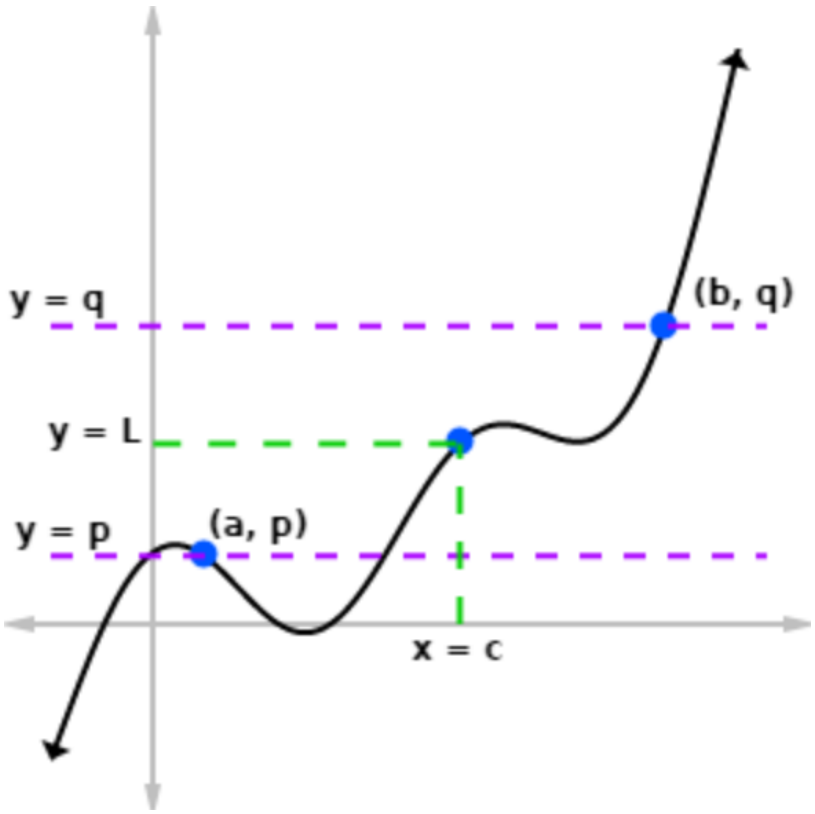
$$f(x)=\begin{cases} x^2-1 & for\ x<3 \\ 2ax & for\ x\geq 3 \end{cases}$$

**VII** For which values of *a* and *b* is the following function *continuous everywhere?*

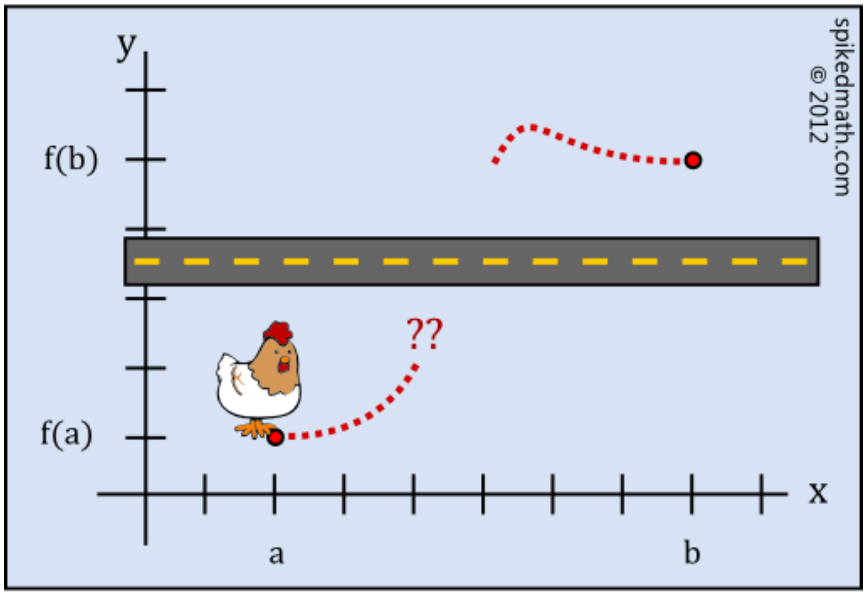
$$g(x)=\begin{cases} ax+2b & for\ x\leq 0 \\ x^2+3a-b & for\ 0<x\leq 2 \\ 3x-5 & for\ x>2 \end{cases}$$

**VIII** State the *Intermediate Value Theorem.* Using the IVT, prove that the polynomial $f(x) = x^4 + 4x^3 - 20x + 11$ must have a root between x = 0 and x = 1 .



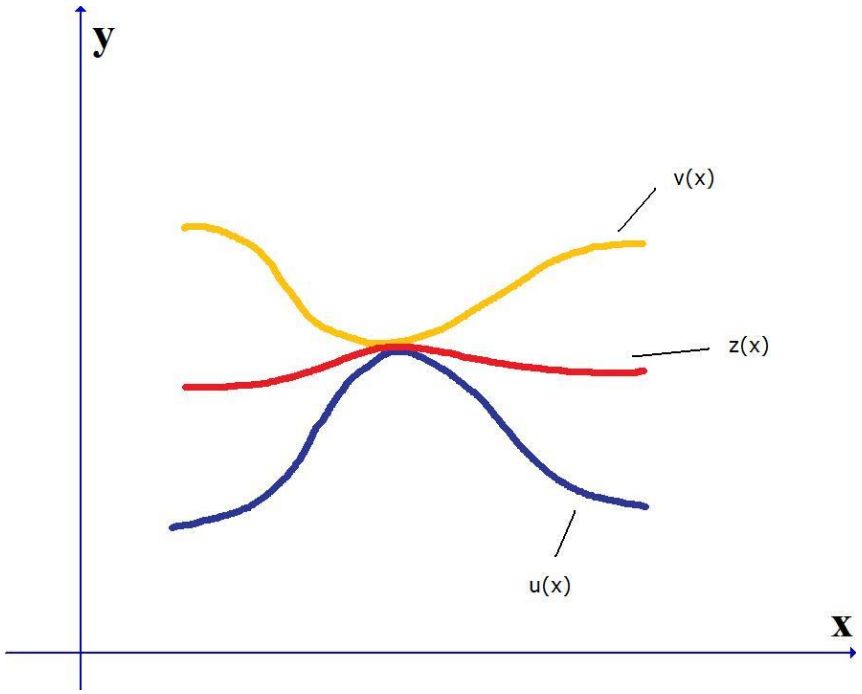


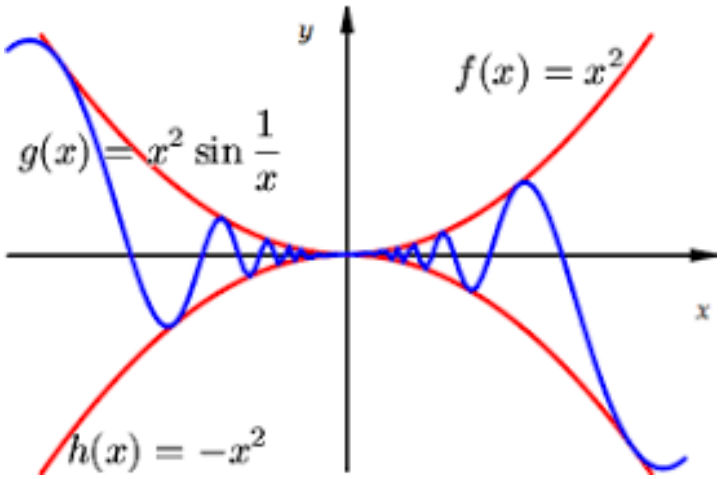

WHY DID THE CHICKEN CROSS THE ROAD?




THE INTERMEDIATE VALUE THEOREM.

**IX** State the *Squeeze Theorem (a.k.a. Sandwich Theorem, Pinching Theorem, Two Gendarmes Theorem, Two Policemen and a Drunk Theorem).*

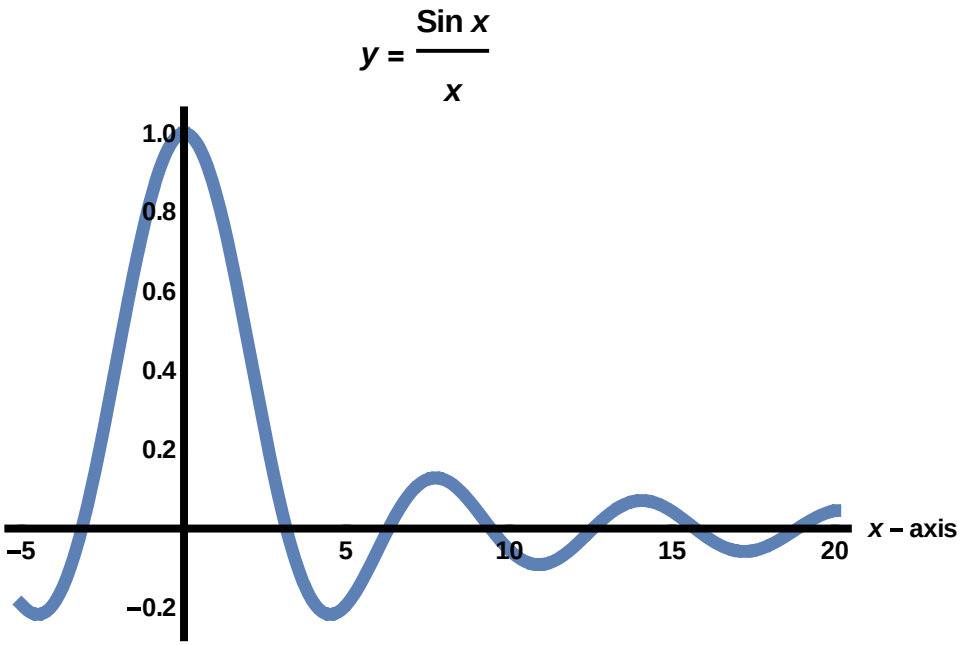


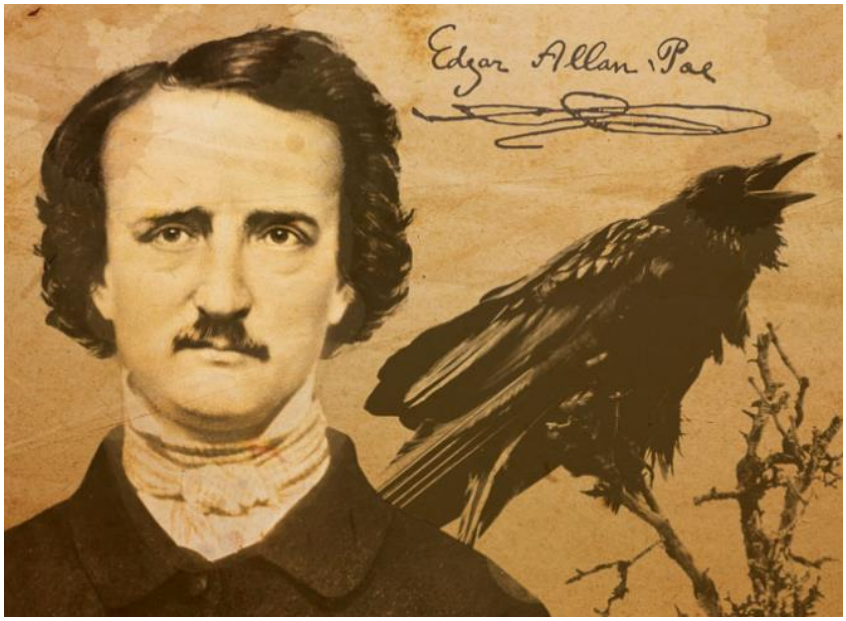




**X** (a) Is the function f(x) = (sin x)/x *even* or *odd* or neither?

(b) Using the Sandwich Theorem prove that $\frac{\sin x}{x} \to 1 \; as \; x \to 0$.

(This result is the key to our being able to differentiate the trig functions.)




“I could have clasped the red walls to my bosom as a garment of eternal peace. "Death," I said, "any death but that of the pit!" Fool! might I have not known that into the pit it was the object of the burning iron to urge me?”

— Edgar Allan Poe, The Pit and the Pendulum