MATH 161 CLASS DISCUSSION 11 SEPT 2017

1. Carefully state the *Squeeze Theorem*. Using the Squeeze Theorem compute each of the following limits:

$$(a)\ \lim_{x\to 0} x^8 \sin^4(1/x)$$

$$(b)\ \lim_{x\to 0} x^4 \cos(1/x)$$

$$(c)\ \lim_{x\to \infty} x\sin(1/x)$$

$$(d)\ \lim_{x\to\infty} \frac{x^2\cos(2x)+\sin^3(x^{2017})}{x^3+x+5}$$

2. (a) State carefully the *Intermediate Value Theorem*.

   (b) Using the Intermediate Value Theorem, explain why the polynomial function $g(x) = x^5 - 4x^3 + 3x - 1$ has at least one real positive root x.

3. Compute $\displaystyle\lim_{x\to 0} \frac{\sin 5x}{\tan 11x}$. Show your work.

4. Compute $\displaystyle\lim_{x\to 0} \frac{\sin(3\cos x)}{\cos(\sin x)}$. Show your work.

5. Carefully state the *Intermediate Value Theorem*. Let $f(x) = 7 + 2x - x^3$ be defined on the interval [1, 3].

   (a) Explain why f must assume the value 0 somewhere on this interval.

   (b) Must f assume the value -13 on the interval [1, 3]? Does the Theorem imply that f must assume the value 9.3 on the interval [1, 3]?

6. Compute $\displaystyle\lim_{x\to 0}\left(\frac{\tan^3 5x}{\tan^3 2x} + x\csc\frac{x}{2} + x\sin\frac{3}{x}\right)$. Show your work.

7. Compute $\displaystyle\lim_{x\to 0}\frac{\sin ax}{\sin bx}$. Have you made any assumptions about the constants a and b?

8. The cost of extracting T tons of ore from a copper mine is $C = F(T)$ dollars. Using a complete sentence that avoids mathematical terminology, explain the meaning of F(2000) = 300,000. (Include appropriate units.)

9. Albertine travels from Chartres to Mt. Saint Michelle at an average speed of 50 km/hr. She returns to Chartres at an average speed of 60 km/hr. What is Albertine's *average speed* during the roundtrip?

10. The expression

$$\frac{V(3) - V(1)}{3 - 1}$$

represents

(a) The average rate of change of the radius with respect to the volume when the radius changes from 1 inch to 3 inches.
(b) The average rate of change of the radius with respect to the volume when the volume changes from 1 cubic inch to 3 cubic inches.
(c) The average rate of change of the volume with respect to the radius when the radius changes from 1 inch to 3 inches.
(d) The average rate of change of the volume with respect to the radius when the volume changes from 1 cubic inch to 3 cubic inches.

11. A paperback book (definitely not a valuable calculus textbook, of course) is dropped from the top of Dennison hall (which is 40 m high) towards a very large, upward pointing fan. The average velocity of the book between time $t = 0$ and later times is shown in the table of data below (in which t is in seconds and the velocities are in m/s).

between $t = 0$ seconds and $t =$	1	2	3	4	5
average velocity is	-5	-10	-11.67	-9	-7.2

a. [8 points] Fill in the following table of values for the height $h(t)$ of the book (measured in meters). Show how you obtain your values.

t	0	1	2	3	4	5
$h(t)$	40	____	____	____	____	____

Which of the following represents the rate at which the volume is changing when the radius is 1 inch?

(a) $\dfrac{V(1.01) - V(1)}{0.01} = 12.69 \text{ in}^3$

(b) $\dfrac{V(0.99) - V(1)}{-0.01} = 12.44 \text{ in}^3$

(c) $\lim_{h \to 0} \left(\dfrac{V(1+h) - V(1)}{h} \right) \text{ in}^3$

(d) All of the above

12.

Which of the following expressions represents the slope of a line drawn between the two points marked in Figure 2.5?

(a) $\dfrac{F(\Delta x) - F(x)}{\Delta x}$ (b) $\dfrac{F(x + \Delta x) - F(x)}{\Delta x}$ (c) $\dfrac{F(x + \Delta x) - F(x)}{x}$

(d) $\dfrac{F(x + \Delta x) - F(x)}{x + x - \Delta x}$ (e) $\dfrac{F(x + \Delta x) - F(x)}{x + \Delta x}$

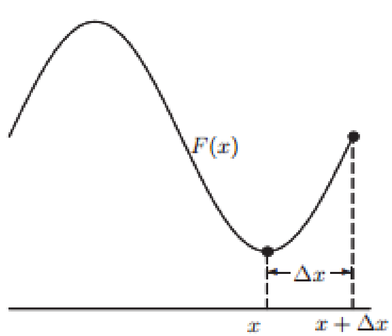

Figure 2.5

13. Suppose that when you merge onto the highway the blue care in from of you is moving at 55 mph. Immediately after you merge, the driver of the blue car speeds up until, after five minutes, it is going 85 mph. Then, during the next five minutes it slows down to 55 mph again. This process then repeats over the following 10 minutes, with the blue car speeding up to 85 mph and then decreasing to 55 mph again.

a. [6 points] Assuming the speed of the blue car follows a sinusoidal pattern, on the axes below draw a well-labeled sketch of two periods of a function $v(t)$ which outputs the speed of the car t minutes after you merge onto the highway.

14. (University of Michigan problem)

A runner competed in a half marathon in Anaheim, a distance of 13.1 miles. She ran the first 7 miles at a steady pace in 48 minutes, the second 3 miles at a steady pace in 28 minutes and the last 3.1 miles at a steady pace in 18 minutes.

a) Sketch a well-labeled graph of her distance completed with respect to time.

b) Sketch a well-labeled graph of her velocity with respect to time.

Everything should be made as simple as possible, but not simpler.

– Albert Einstein, **Reader's Digest** (Oct. 1977)