MATH 161 PRACTICE FINAL EXAM A

PART I (*6 pts each*) ***Answer any 17 of the following 21 questions.*** You need not justify your answer. You may answer more than 17 to obtain extra credit.

1. $\lim_{n\to\infty} \dfrac{(2n+1)^3(n+2017)^5}{n(4n+3)(n-2017)^7}$

2. $\lim_{x\to 0} \dfrac{\cos x - 1}{e^x - x^2 - x - 1}$

3. $\lim_{x\to 0} \dfrac{\ln(ax+1)}{\ln(bx+1)}$ where a and b are positive constants.

4. Let $h(x) = \int_1^x \ln\left(1 + 2017 \ln t\right) dt$

   Compute $h'(\mathrm{e})$.

5. $\dfrac{d^{2017}}{dx^{2017}} \sin(7x) =$

6. Solve the initial value problem:

   $\dfrac{dy}{dx} = \dfrac{\ln x}{x}$ given that y = 2017 when x = 1.

7. Find an anti-derivative of $\dfrac{\left(1+\sqrt{x}\right)^{\frac{4}{5}}}{\sqrt{x}}$

8. Find an anti-derivative of:

$$\frac{1+3e^{3x}-e^{-x}}{e^{3x}+e^{-x}+x}$$

9. $\lim\limits_{x\to\infty} x\tan\left(\frac{1789}{x}\right)=$

10. Suppose that $\int_7^{13} f(x)\,dx = 3$ *and* $\int_7^{13} g(x)\,dx = 1.$

$$Find \int_7^{13}\left(4f(x)-3g(x)+2\right)dx$$

11. Find the *average value* of the function y = $\sec^2$ x over the interval [0, π/4]. (Give the precise result without rounding.)

12. Find the value of *c* such that the conclusion of the Mean Value Theorem is verified for the function $g(x)=\frac{1}{(x-1)^2}$ on the interval [2, 5]. Express your answer to the nearest hundredth.

13. *Find* $\lim\limits_{n\to\infty}\left(\sqrt{n^2+92n+2017}\; - n\right)$

14. Let *a* and *b* be non-zero constants. *Let* $f(x)=\frac{x+a}{x^2+b}$.

Find the *slope of the tangent line* to y = f(x) at x = 0. (Your answer may include the constants *a* and *b*.)

15. Let *a* and *b* be non-zero constants. Then $\int \frac{\sec x\tan x}{a+b\sec x}\,dx=$

16. Suppose that $\int_1^x g(t)\,dt = x^3-1.$ Find g(5).

17. *Let* $y=x^{x^2+x+1}$. Find dy/dx when x = 1.

18. *Compute* $\int_{-1}^{3} |x|\,dx.$

19. *Compute* $\lim\limits_{n\to\infty}\sum_{j=1}^{n}\frac{1}{n}\sin\left(\frac{j}{n}\pi\right)$ (*Hint:* Convert this limit into a Riemann integral.)

20. *Given that* $G(x) = \int_0^{3x^2} \sqrt{1+t^3}\,dt$, *find* $G'(1)$.

21. Charlotte, the spider, lives on the x-axis. Suppose that at time t = 1 minute, she is at x = 5 cm, and that her velocity (in cm/minute) at time *t* is given by:
$v(t) = 4t^3 - 6t^2 + 1$. *Where* is Charlotte at time *t = 2 minutes*?

PART II *(12 pts each)*

Answer any 13 of the following 14 problems. You may answer more than 11 for extra credit.

1. Find the equation of the *tangent line* to the curve defined implicitly by

$$x^4 + y^3 - x^2 y = 13 + \ln y$$

at the point P = (2, 1).

2. Gilberte, who is 5 feet tall, walks away from an 18 foot lamppost. She observes that when she is 8 feet from the base of the lamppost, her shadow is increasing at a rate of 6 ft/min. Find Gilberte's speed when she is 8 feet from the base of the lamppost.

3. Using an *appropriate tangent line approximation*, estimate the value of $\sqrt[5]{1.0004}$. Have you obtained an *overestimate* or an *underestimate*? Explain. Sketch!

4. We wish to approximate a root of $g(x) = x^4 + x - 1$. Note that $g(0) < 0$ and $g(1) > 0$.

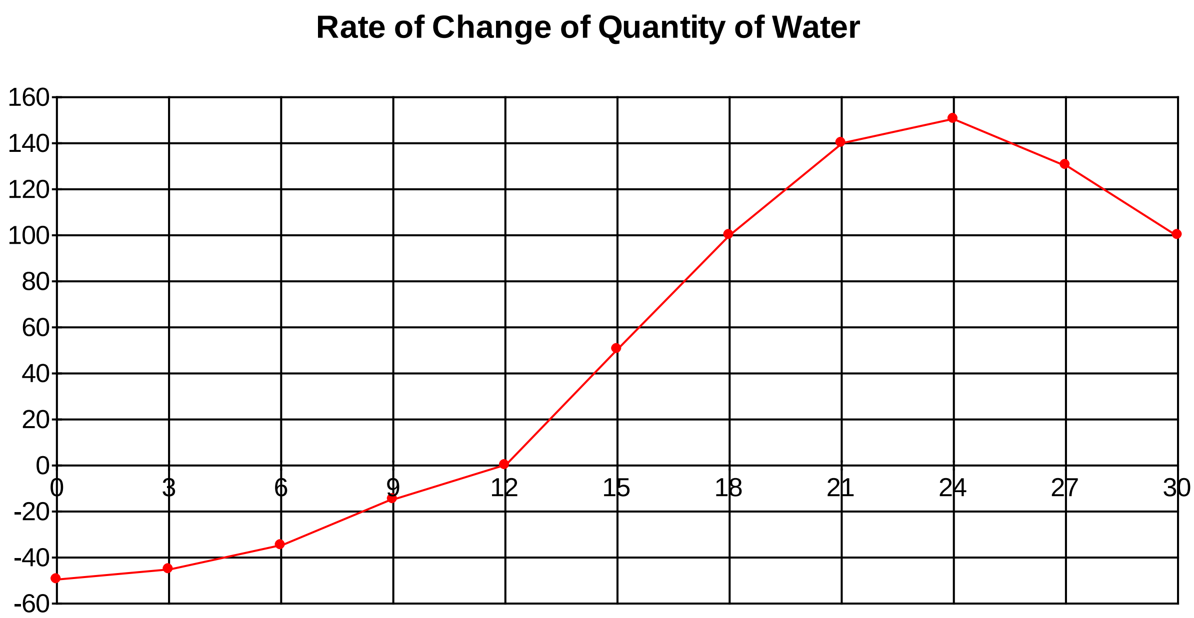
   (a) How do we know that there must exist a solution to $g(x) = 0$ in the interval (0, 1)?

   (b) Let our initial guess for the root be $x_0 = 0.5$. Using Newton's method, compute x_1 and x_2 (to at least 3 significant digits).

5. Graph the function $f(x) = (x-1)^2 e^x$. Identify any and all local and global extrema and points of inflection.

6. Madam Verdurin is building an open planter in the shape of a rectangular box with a square base. The base is made of metal that costs $7 per square foot. The sides are made of wood that costs $3 per square foot. The planter must hold at least 8 cubic feet of dirt. Find the dimensions of the *least expensive* planter that Madame Verdurin can build.

7. The graph below shows the *RATE OF CHANGE* of the quantity of water in the Water Tower of OZ, in liters per day, during the month of April, 2009. The tower contained 12,000 liters of water on April 1. *Estimate* the quantity of water in the tower on April 30. Show your work.



8. Using the FTC, find the area bounded by the two parabolas:

$y = x^2 - 5x$ and $y = 20 + x - x^2$. *Sketch.*

9. Use a *left-endpoint* Riemann sum with $n = 4$ rectangles to approximate the area under the curve $f(x) = \dfrac{1}{x^3 + 1}$ between x = 0 and x = 2. Draw a picture to illustrate what you are computing. Is this an *underestimate* or an *overestimate* of the area? *Explain!*

10. The function y = F(x) is defined below:

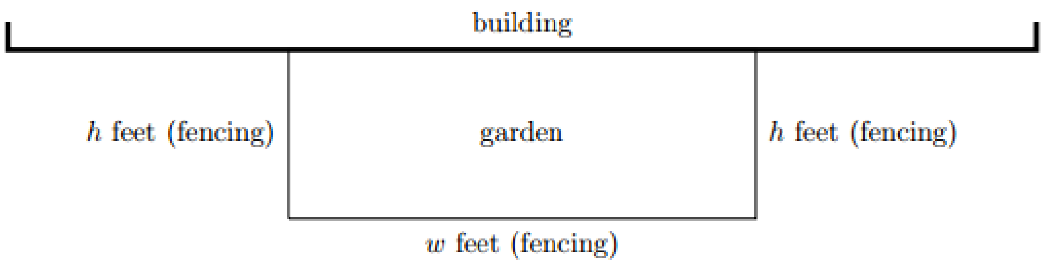
$$F(x) = \begin{cases} \dfrac{3x^4 - 2x^3 - 21x^2}{x-3} & \text{if } x \neq 3 \\ k & \text{if } x = 3 \end{cases}$$

For which value(s) (if any) of *k* is the function everywhere continuous? Explain!

11. Graph the cubic polynomial $g(x) = x^3 + x^2 - 8x + 5$. Identify any and all local and global extrema and points of inflection.

12. *(University of Michigan)*

[12 points] Researchers are constructing a rectangular garden adjacent to their building. The garden will be bounded by the building on one side and by a fence on the other three sides. (See diagram below.) The fencing will cost them \$5 per linear foot. In addition, they will also need topsoil to cover the entire area of the garden. The topsoil will cost \$4 per square foot of the garden's area.
Assume the building is wider than any garden the researchers could afford to build.



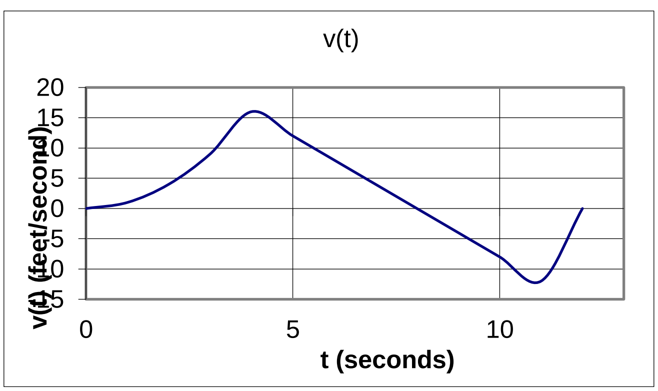
a. [5 points] Suppose the garden is w feet wide and extends h feet from the building, as shown in the diagram above. Assume it costs the researchers a total of \$250 for the fencing and topsoil to construct this garden. Find a formula for w in terms of h.

b. [3 points] Let $A(h)$ be the total area (in square feet) of the garden if it costs \$250 and extends h feet from the building, as shown above. Find a formula for the function $A(h)$. The variable w should not appear in your answer.

(Note that $A(h)$ is the function one would use to find the value of h maximizing the area. You should not do the optimization in this case.)

c. [4 points] In the context of this problem, what is the domain of $A(h)$?

13. Albertine launches a model rocket from the ground at time t = 0. The rocket starts by traveling straight up in the air. The graph below illustrates the upward velocity of the rocket as a function of time.

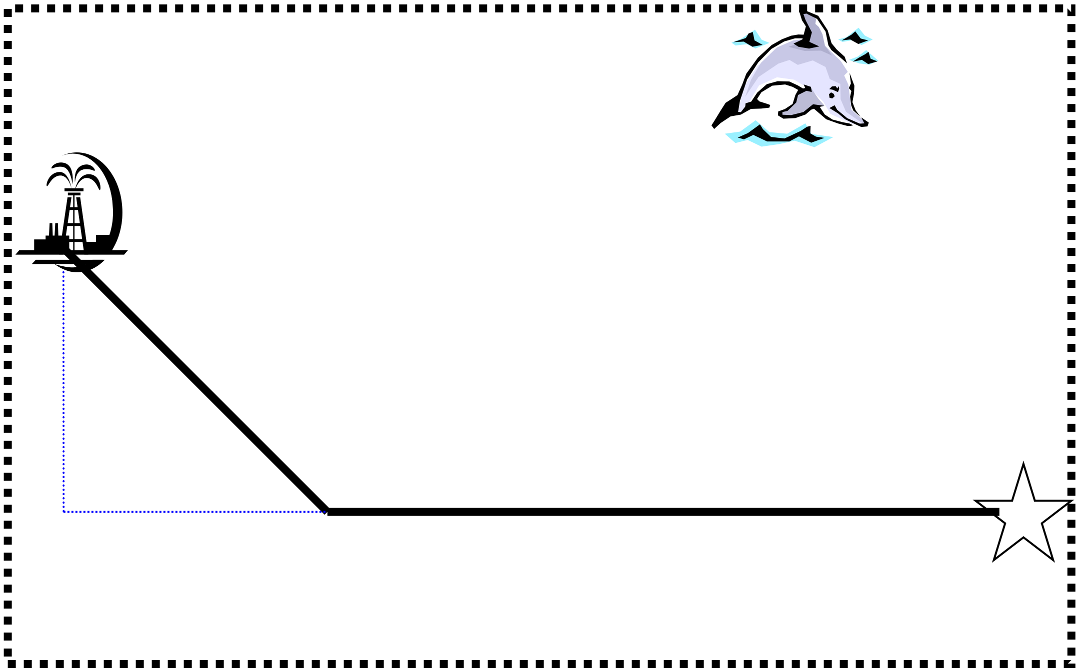


(a) Sketch a graph of the *acceleration* of the rocket as a function of time.

(b) Sketch a graph of the *height* of the rocket as a function of time.

(c) Give an estimate of the *maximum height* the rocket achieved.

14. Oil from an offshore rig located 3 miles from the shore is to be pumped to a location on the edge of the shore that is 9 miles east of the rig. The cost per foot of constructing a pipe in the ocean from the rig to the shore is *twice the cost per foot* of construction on land. Determine how the pipe should be laid to *minimize the total cost*.



With an absurd oversimplification, the "invention" of the calculus is sometimes ascribed to two men, Newton and Leibniz. In reality, the calculus is the product of a long evolution that was neither initiated nor terminated by Newton and Leibniz, but in which both played a decisive part.

- Richard Courant and Herbert Robbins

The quarrel [between Newton and Leibniz] is simply the expression of evil weaknesses and fostered by vile people. Just what would Newton have lost if he had acknowledged Leibniz's originality? Absolutely nothing! He would have gained a lot. And yet how hard it is to acknowledge something of this sort: someone who tries it feels as though he were confessing his own incapacity. ... It's a question of envy of course. And anyone who experiences it ought to keep on telling himself: "It's a mistake! It's a mistake! -- "

- Ludwig Wittgenstein (1947)