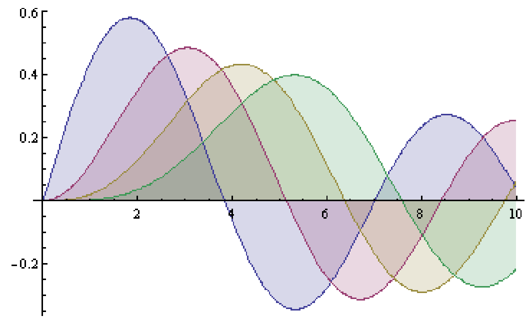
MATHEMATICA LAB III



AREA AND THE RIEMANN INTEGRAL

Due: 3rd December 2018

Submit a *printed version* of your Mathematica notebook. You may work with other students and compare results, but ultimately you must submit *your own* lab results --- not a shared copy. On your front page (using Mathematica) state your name and "Mathematica Lab III" using an appropriate style, font, size, and color. *Before solving each problem, state the problem. Remember to staple!*

I For each of the following area problems, begin by graphing the curves to see what they look like and how many points of intersection there are. Use FindRoot to find the points of intersection. The area between *f* and *g* over the interval [a, b] equals

NIntegrate[Abs[f [x]-g[x]], {x, a, b}].

(a) Find the area between the curve $g(x) = x^4 - 15x^3 + 54x^2 + 26x - 257$ and the x-axis.

(b) Find the area between the curves $y = 2\cos(9x)$ and $y = 5x$.

(c) Find the area between the curves $y = x + \sin(2x)$ and $y = x^3$.

(d) Find the area between the curves $y = x^2\cos x$ and $y = x^3 - x$.

II (This exercise is due to G. Thomas.) Karl Weierstrass' example of a continuous function that is *nowhere* differentiable is given by an infinite series

$$\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n \cos(9^n \pi x).$$

Infinite series will be explored in Math 162. However, we can learn a great deal about an infinite series by examining its first few terms. In the case of the famous Weierstrass example, let

$$F_K(x) = \cos(\pi x) + \frac{2}{3}\cos(9\pi x) + \left(\frac{2}{3}\right)^2 \cos(9^2\pi x) + \left(\frac{2}{3}\right)^3 \cos(9^3\pi x) + \cdots \left(\frac{2}{3}\right)^K \cos(9^K\pi x)$$

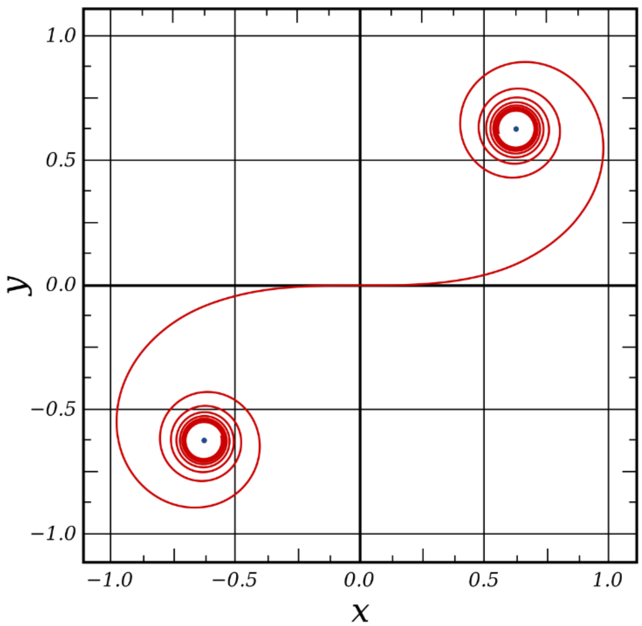
(a) Plot F_K for *several* small values of K (say, K = 1, 2, 3, 4, 5) for a suitable domain. Observe how the graph of F_K is both "wiggly" and "bumpy."

(b) Next, graph the *derivative of f* on another set of axes. Make a couple of observations.

III (Stewart) The functions, $S(x)$ *and* $C(x)$*, are defined as follows:*

$$S(x) = \int_0^x \sin\left(\frac{1}{2}\pi t^2\right) dt \quad and \quad C(x) = \int_0^x \cos\left(\frac{1}{2}\pi t^2\right) dt$$

S(*x*) and *C*(*x*) are transcendental functions used in optics; they are named after the physicist Augustin-Jean Fresnel and arise in the description of near-field diffraction phenomena.



The **Euler spiral** (on the left) is the curve generated by a parametric plot of S(x) against C(x).

(a) Plot the graphs of y = S(x) and y = C(x) on the same pair of axes. How do they appear to be related?

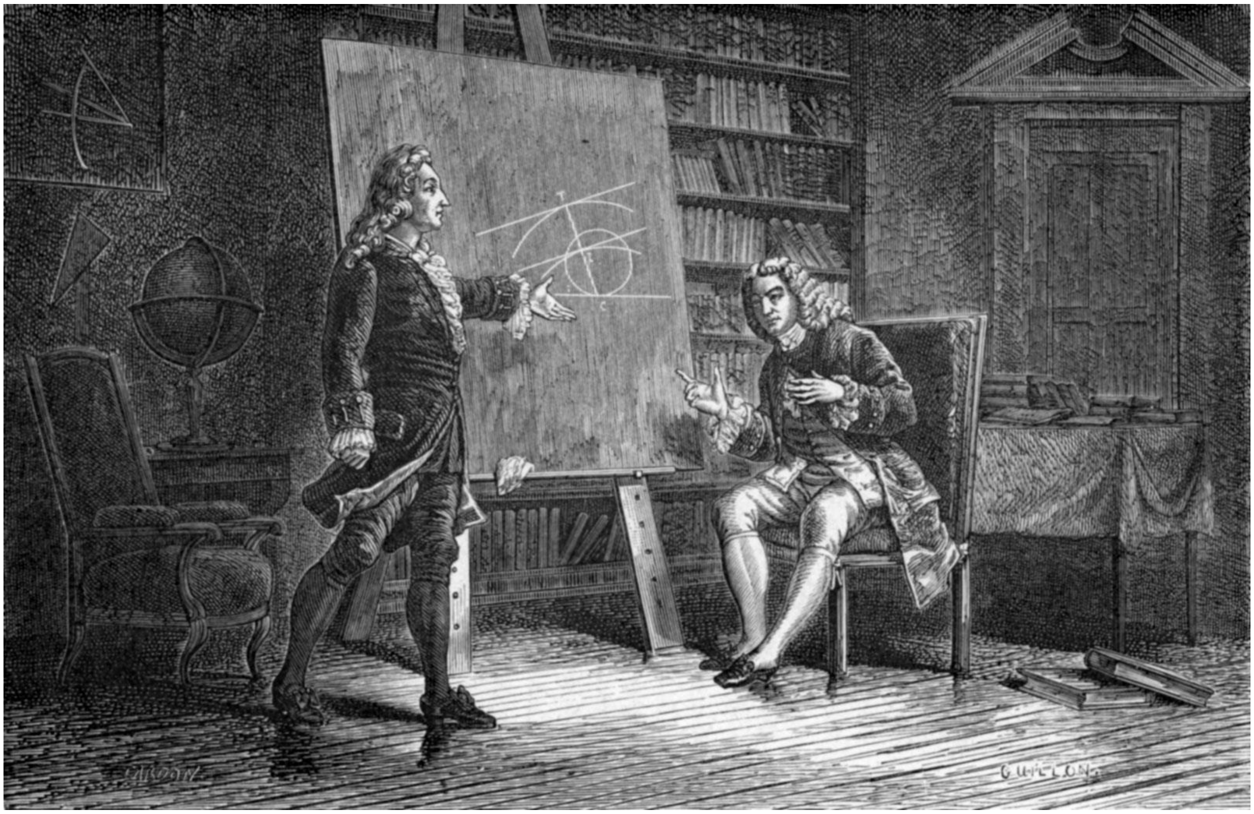
(b) On which intervals is *C* increasing?

(c) On which intervals is *C* concave up?

(d) Solve the equation $\int_0^x \cos\left(\frac{1}{2}\pi t^2\right) dt$ =0.7 correct to two decimal places.

But just as much as it is easy to find the differential of a given quantity, so it is difficult to find the integral of a given differential. Moreover, sometimes we cannot say with certainty whether the integral of a given quantity can be found or not.

– Johann Bernoulli



Johann and Jacob Bernoulli working together