# MATH 161 Answers: TEST I

# September 2017

1. Albertine orders a large cup of coffee at Metropolis on Granville. Let F(t) be the temperature in *degrees Fahrenheit* of her coffee *t* *minutes* after the coffee is placed on her tray.

(a) *(2 pts)* Explain the meaning of the statement: F(9) = 167. (Use complete sentences. Avoid any mathematical terms!)



(b) *(3 pts)* Explain the meaning of the statement: F-1(99) = 17.5

(c) *(3 pts)* Give the *practical* interpretation of the statement: F′(9) = – 1.10. (Use complete sentences. Do not use the words “derivative” or “rate” or any other mathematical term in your explanation.)

(d) *(1 pt)* What are the *units* of F′(9)?

*Answer: F/ minute (since F/ t represents change in temp/ change in time.)*

(e) *(3 pts)* Using the information given in parts (a) and (c), estimate the temperature of Albertine’s coffee *seven* minutes after she has been handed the coffee.

*Answer:* ***169.2 F****.*

 (f) *(3 pts)* Extra Credit

Explain the meaning of the statement:

 $\left(F^{-1}\right)^{'}\left(99\right)=-1$

1. *(4 pts each)* For each of the following three sets of axes, exactly one of the following statements (a) – (e) is true. You may use a letter more than once. In the space provided next to each figure, enter the letter of the true statement for that figure. For each graph, note that the x and y scales are not the same.





*True Statement: A*



*True Statement: E*



*True Statement: D*

1. *(12 pts)* Using the limit definition of the derivative, write an explicit expression for the *derivative* of the function

g(x) = (cos x)x at x = 3.

*Do not try to calculate this derivative.*

*Answer:*

$$ g^{'}\left(3\right)=\lim\_{h\to 0}\frac{g\left(h+3\right)-g(3)}{h}=\lim\_{h\to 0}\frac{\left(\cos((h+3))\right)^{h+3}-(\cos(3))^{3}}{h}$$

1. *(a) (6 pts)* *Find* $\lim\_{x\to \infty }f\left(x\right) if, for all x>5, $

*Explain!*

*Which theorem are you using?*

*Answer: Use the* ***Squeeze Theorem*** *to conclude that f(x)*$ \rightarrow 4$$as x\rightarrow \infty .$

(b) *(6 pts)* Show that y = f(x) = x3 + 5ex + 1 has *at least one* real root. *Explain!*

Which theorem are you using?

*Answer: Using the* ***Intermediate Value Theorem*** *we may conclude that f must have a root on the interval*

 *(-10, 1). (Of course, this is only one of infinitely many correct answers.)*

5. *(12 pts)* Find an equation of the *normal line* to the curve

$y=g\left(x\right)=\frac{x^{2}-1}{x^{2}+1}$ at x = 1.

You may use shortcuts.

*Answer:* ***y = 1 – x****.*

6. *(12 pts)* The graph of a function *g* is shown below. For each of the following, decide if the limit exists. If it does, find the limit. If it does not, decide if the “limit” is ∞, -∞, or neither. No justification is necessary for full credit, but show your work for purposes of partial credit.



*Answers:*

1. $ \lim\_{x\to 0^{+}}g(x)=2 $
2. $\lim\_{x\to 1^{-}}g(x)$*=2*
3. $\lim\_{x\to 1^{+}}g(x)$*=1*
4. $\lim\_{x\to 1}g(x)$*=does not exist; neither ± ∞.*
5. $\lim\_{x\to 2}g(x)$*= 0*
6. $\lim\_{x\to 0^{+}}g\left(x\right)= $ *[redundant]*
7. $\lim\_{x\to 3^{-}}g(x)$*=1*
8. $\lim\_{x\to 3^{+}}g\left(x\right)= -1$
9. $\lim\_{x\to 3}g\left(x\right)= $*does not exist; neither ± ∞.*
10. $\lim\_{x\to 4^{+}}g\left(x\right)= $*∞*
11. $\lim\_{x\to 4^{-}}g\left(x\right)= $ *-∞*
12. $\lim\_{x\to 4}g(x)= $*does not exist; neither ± ∞.*
13. $\lim\_{x\to 5^{-}}g(x)= $*1*

7. Suppose that f(x) is a function that is continuous on the interval [-2, 2]. The graph of $f^{'}\left(x\right)on the interval \left[-2, 2\right] is given below.$



1. *(6 pts)* Let y = L(x) be the local linearization of f(x) at x = -1. Using the fact that f(-1) = -4, write a formula for y = L(x).

*Answer: L(x) = -4+3(x+1)*

1. *(6 pts)* Use your formula for L(x) to approximate f(-0.5).

*Answer:* ***-5.5***

8. Suppose that *f* and *g* are differentiable functions satisfying:

f(3) = -2, g(3) = -4, f ′(3) = 3, and g′(3) = -1.

1. *(6 pts)* Let H(x) = (f(x) + 2g(x) + 1)(f(x) – g(x) – 4). Compute H′(3) (Hint: Use short cuts here.)

*Answer:* ***-38***

 Compute M′(3)

*Answer:*  $\frac{45}{98}$

9. For each of the following, find any and all critical points. Then, using the first derivative test, classify them (local max, local min, neither).

1. *(6 pts)* y = f(x) = x3 – 3x + 1

*Answer: f achieves a* ***local maximum*** *at x = -1 and a* ***local minimum*** *at x =1.*

1. *(6 pts)* y = g(x) = 3x4 –16x3 + 18x2 + 1

*Answer: g achieves a* ***local maximum*** *at x = 4 and* ***local minima*** *at x = 1 and x = 6.*

10. Let y = f(x) be a differentiable function with derivative

$$f^{'}\left(x\right)= \frac{e^{x}(x-1)\left(x-2\right)^{2}\left(x-4\right)^{3}\left(x-5\right)^{4}(x-6)^{5}}{1+x^{4}}$$

1. *(4 pts)* Find any and all critical points.

$$Answer: x=1, 2, 4, 5, and 6$$

1. *(8 pts)* Classify each critical point (local max, local min, neither).

*Answer: Using the first derivative test, we find that x = 4 is a local maximum; x= 1 and x = 6 are both local minima.*

11. *(3 pts each)* Compute each of the following limits. *Justify your reasoning.*



*Answer:* ***27/2***



Answer: - 2/27

 

*Answer: 0*



*Answer:* ***½***

12. *(3 pts each)* For each of the following functions, determine the type of discontinuity at the given point. If it is a *removable* discontinuity, find a continuous extension of the function.

1. $y= \frac{x^{3}-x^{2}-2x}{\left(x-2\right)(x+5)}$ at x = 2

*Answer: This is a* ***removable discontinuity****. The continuous extension requires that y be defined as* $\frac{6}{7}$ *when x = 2.*

1. $y= \frac{x^{3}-x^{2}-2x}{\left(x-2\right)(x+5)}$ at x = -5

*Answer: an* ***infinite discontinuity****.*

1. $y= cos\frac{3}{x} at x=0$

*Answer:* ***essential discontinuity****.*

1. $y= \frac{|x|}{x} at x=0$

*Answer:* ***jump discontinuity***