MATH 161 ANSWERS: TEST I

SEPTEMBER 2017

1. Albertine orders a large cup of coffee at Metropolis on Granville. Let F(t) be the temperature in *degrees Fahrenheit* of her coffee *t minutes* after the coffee is placed on her tray.

(a) *(2 pts)* Explain the meaning of the statement: F(9) = 167. (Use complete sentences. Avoid any mathematical terms!)

(b) *(3 pts)* Explain the meaning of the statement: $F^{-1}(99) = 17.5$

(c) *(3 pts)* Give the *practical* interpretation of the statement: $F'(9) = -1.10$. (Use complete sentences. Do not use the words "derivative" or "rate" or any other mathematical term in your explanation.)

(d) *(1 pt)* What are the *units* of $F'(9)$?

Answer: °F/ minute (since $\Delta F/\Delta t$ represents change in temp/ change in time.)

(e) *(3 pts)* Using the information given in parts (a) and (c), estimate the temperature of Albertine's coffee *seven* minutes after she has been handed the coffee.

Answer: ***169.2 °F****.*

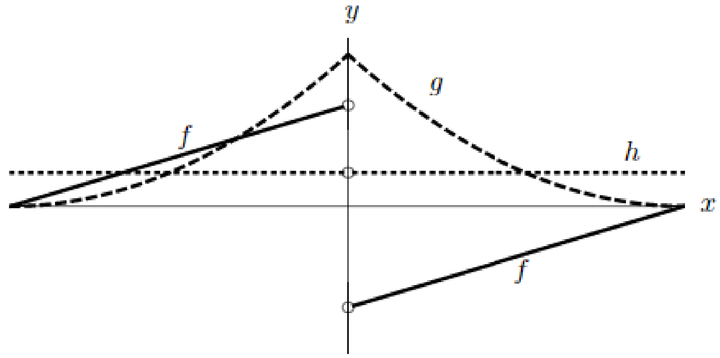
(f) *(3 pts)* EXTRA CREDIT

Explain the meaning of the statement:

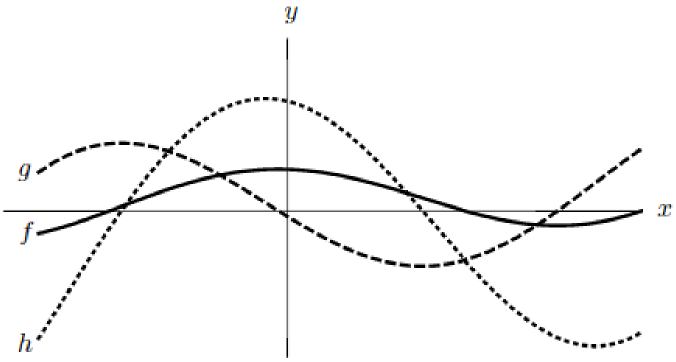
$$(F^{-1})'(99) = -1$$

2. *(4 pts each)* For each of the following three sets of axes, exactly one of the following statements (a) – (e) is true. You may use a letter more than once. In the space provided next to each figure, enter the letter of the true statement for that figure. For each graph, note that the x and y scales are not the same.

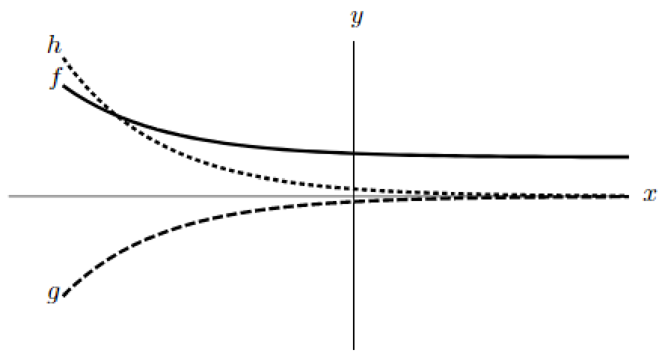
(a) h is the derivative of f, and f is the derivative of g.
(b) g is the derivative of f, and f is the derivative of h.
(c) g is the derivative of h, and h is the derivative of f.
(d) h is the derivative of g, and g is the derivative of f.
(e) None of **(a)**-**(d)** are possible.



True Statement: A



True Statement: E



True Statement: D

3. *(12 pts)* Using the limit definition of the derivative, write an explicit expression for the *derivative* of the function

$$g(x) = (\cos x)^x \text{ at } x = 3.$$

Do not try to calculate this derivative.

Answer:

$$g'(3) = \lim_{h\to 0} \frac{g(h+3) - g(3)}{h} = \lim_{h\to 0} \frac{(\cos(h+3))^{h+3} - (\cos 3)^3}{h}$$

4. *(a) (6 pts)* *Find* $\lim_{x\to\infty} f(x)$ *if, for all* $x > 5$,

$$\frac{4x-1}{x} < f(x) < \frac{4x^2+3x}{x^2}$$

Explain!

Which theorem are you using?

*Answer: Use the **Squeeze Theorem** to conclude that f(x) → 4 as x → ∞.*

(b) *(6 pts)* Show that y = f(x) = $x^3 + 5e^x + 1$ has *at least one* real root. *Explain!*

Which theorem are you using?

*Answer: Using the **Intermediate Value Theorem** we may conclude that f must have a root on the interval (-10, 1). (Of course, this is only one of infinitely many correct answers.)*

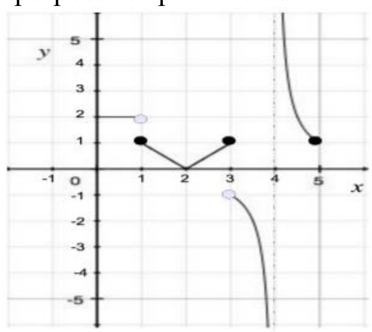
5. *(12 pts)* Find an equation of the *normal line* to the curve

$$y = g(x) = \frac{x^2-1}{x^2+1} \text{ at } x = 1.$$

You may use shortcuts.

Answer: ***y = 1 – x****.*

6. *(12 pts)* The graph of a function *g* is shown below. For each of the following, decide if the limit exists. If it does, find the limit. If it does not, decide if the "limit" is ∞, -∞, or neither. No justification is necessary for full credit, but show your work for purposes of partial credit.



Answers:

(a) $\lim_{x\to 0^+} g(x) = 2$

(b) $\lim_{x\to 1^-} g(x) = 2$

(c) $\lim_{x\to 1^+} g(x) = 1$

(d) $\lim_{x\to 1} g(x) =$ *does not exist; neither ± ∞.*

(e) $\lim_{x\to 2} g(x) = 0$

(f) $\lim_{x\to 0^+} g(x) =$ *[redundant]*

(g) $\lim_{x\to 3^-} g(x) = 1$

(h) $\lim_{x\to 3^+} g(x) = -1$

(i) $\lim_{x\to 3} g(x) =$ *does not exist; neither ± ∞.*

(j) $\lim_{x\to 4^+} g(x) = \infty$

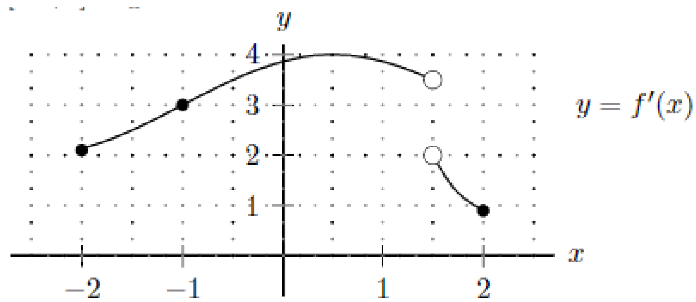
(k) $\lim_{x\to 4^-} g(x) = -\infty$

(l) $\lim_{x\to 4} g(x) =$ *does not exist; neither ± ∞.*

(m) $\lim_{x\to 5^-} g(x) = 1$

7. Suppose that f(x) is a function that is continuous on the interval [-2, 2]. The graph of

$f'(x)$ *on the interval* $[-2, 2]$ *is given below.*



(a) *(6 pts)* Let y = L(x) be the local linearization of f(x) at x = -1. Using the fact that f(-1) = -4, write a formula for y = L(x).

Answer: L(x) = -4+3(x+1)

(b) *(6 pts)* Use your formula for L(x) to approximate f(-0.5).

Answer: ***-5.5***

8. Suppose that *f* and *g* are differentiable functions satisfying:

f(3) = -2, g(3) = -4, f ′(3) = 3, and g′(3) = -1.

(a) *(6 pts)* Let H(x) = (f(x) + 2g(x) + 1)(f(x) – g(x) – 4). Compute H′(3) (Hint: Use short cuts here.)

Answer: ***-38***

(b) (6 *pts*) *Let* $M(x) = \dfrac{2f(x)+3g(x)}{2-3g(x)}$. Compute M′(3)

Answer: $\mathbf{\frac{45}{98}}$

9. For each of the following, find any and all critical points. Then, using the first derivative test, classify them (local max, local min, neither).

(a) *(6 pts)* y = f(x) = $x^3 - 3x + 1$

Answer: f achieves a ***local maximum*** *at x = -1 and a* ***local minimum*** *at x =1.*

(b) *(6 pts)* y = g(x) = $3x^4 - 16x^3 + 18x^2 + 1$

Answer: g achieves a ***local maximum*** *at x = 4 and* ***local minima*** *at x = 1 and x = 6.*

10. Let y = f(x) be a differentiable function with derivative

$$f'(x) = \frac{e^x(x-1)(x-2)^2(x-4)^3(x-5)^4(x-6)^5}{1+x^4}$$

(a) *(4 pts)* Find any and all critical points.

Answer: $x = 1, 2, 4, 5, and\ 6$

(b) *(8 pts)* Classify each critical point (local max, local min, neither).

Answer: Using the first derivative test, we find that x = 4 is a local maximum; x= 1 and x = 6 are both local minima.

11. *(3 pts each)* Compute each of the following limits. *Justify your reasoning.*

$$(a) \quad \lim_{x\to\infty} \frac{(4x^3+11)^2(3x-91)^3}{(2x^2+5)^4(2x+2017)}$$

Answer: ***27/2***

$$(b) \quad \lim_{x\to 3} \frac{\frac{1}{x^2}-\frac{1}{9}}{x-3}$$

Answer: - 2/27

$$(c) \quad \lim_{x\to\infty} \frac{\sin 7x}{x}$$

Answer: 0

$$(d) \quad \lim_{x\to 0} \frac{\sqrt{x+1}-1}{x}$$

Answer: ½

12. *(3 pts each)* For each of the following functions, determine the type of discontinuity at the given point. If it is a *removable* discontinuity, find a continuous extension of the function.

(a) $y = \frac{x^3-x^2-2x}{(x-2)(x+5)}$ at x = 2

Answer: This is a ***removable discontinuity****. The continuous extension requires that y be defined as* $\frac{6}{7}$ *when x = 2.*

(b) $y = \frac{x^3-x^2-2x}{(x-2)(x+5)}$ at x = -5

Answer: an ***infinite discontinuity****.*

(c) $y = \cos\frac{3}{x} \quad at\ x = 0$

Answer: ***essential discontinuity****.*

(d) $y = \frac{|x|}{x} \quad at\ x = 0$

Answer: ***jump discontinuity***