# MATH 161 Solutions: TEST I

# September 2017

**Instructions:** *Answer any 10 of the following 12 questions. You may solve more than 10 to obtain extra credit.* ☺

 1. Albertine orders a large cup of coffee at Metropolis on Granville. Let F(t) be the temperature in *degrees Fahrenheit* of her coffee *t* *minutes* after the coffee is placed on her tray.

(a) *(2 pts)* Explain the meaning of the statement: F(9) = 167. (Use complete sentences. Avoid any mathematical terms!)



*Solution: Nine minutes after the cup of coffee is placed upon the tray, Albertine finds that the temperature of the coffee is 167 F.*

(b) *(3 pts)* Explain the meaning of the statement: F-1(99) = 17.5

*Solution: When the temperature of the coffee is 99 F, 17.5 minutes have elapsed since the coffee mug was placed upon Albertine’s tray.*

(c) *(3 pts)* Give the *practical* interpretation of the statement: F′(9) = – 1.10. (Use complete sentences. Do not use the words “derivative” or “rate” or any other mathematical term in your explanation.)

*Solution: Nine minutes after being served her coffee, Albertine notices that the temperature of the coffee is decreasing by about 1.1 F every minute during the next couple of minutes.*

(d) *(1 pt)* What are the *units* of F′(9)?

*Answer: F/ minute (since F/ t represents change in temp/ change in time.)*

(e) *(3 pts)* Using the information given in parts (a) and (c), estimate the temperature of Albertine’s coffee *seven* minutes after she has been handed the coffee.

*Solution: Since after nine minutes the temperature is 167 F, we would estimate that two minutes earlier the temperature was 167 + 2(1.1) =* ***169.2 F****.*

 (f) *(3 pts)* Extra Credit

Explain the meaning of the statement:

*Solution: When the temperature of the coffee is 99 ºF, time increases by one minute for each ºF that the temperature of the coffee falls.*

1. *(4 pts each)* For each of the following three sets of axes, exactly one of the following statements (a) – (e) is true. You may use a letter more than once. In the space provided next to each figure, enter the letter of the true statement for that figure. For each graph, note that the x and y scales are not the same.





*True Statement: A*



*True Statement: E*



*True Statement: D*

1. *(12 pts)* Using the limit definition of the derivative, write an explicit expression for the *derivative* of the function

g(x) = (cos x)x at x = 3.

*Do not try to calculate this derivative.*

*Solution:*

1. *(a) (6 pts)* *Find*



*Explain!*

*Which theorem are you using?*

*Solution: Since we can invoke the* ***Squeeze Theorem*** *to conclude that f(x)*

(b) *(6 pts)* Show that y = f(x) = x3 + 5ex + 1 has *at least one* real root. *Explain!*

Which theorem are you using?

*Solution: Note that y = f(x) is continuous on the real line since it is a sum of continuous functions.*

*Next, notice that f(1) > 0 and f(-10) = -8.99 < 0.*

*Invoking the* ***Intermediate Value Theorem****, we conclude that f must have a root on the interval (-10, 1).*

5. *(12 pts)* Find an equation of the *normal line* to the curve

 at x = 1.

You may use short cuts.

*Solution: Let y = g(x). Then .*

*Thus.*

*Also note that g(0) = 0, so the point of tangency is (1, 0).*

*Finally, the slope of the normal line is -1 and the equation of the normal line is y – 0 = -(x – 1); more simply,* ***y = 1 – x****.*

6. *(12 pts)* The graph of a function *g* is shown below. For each of the following, decide if the limit exists. If it does, find the limit. If it does not, decide also if the “limit” is ∞, -∞, or neither. No justification is necessary for full credit, but show your work for purposes of partial credit.



1. *=2*
2. *=1*
3. *=does not exist; neither ± ∞.*
4. *= 0*
5. *[redundant]*
6. *=1*
7.
8. *does not exist; neither ± ∞.*
9. *∞*
10. *-∞*
11. *does not exist; neither ± ∞.*
12. *1*

7. Suppose that f(x) is a function that is continuous on the interval [-2, 2]. The graph of



1. *(6 pts)* Let y = L(x) be the local linearization of f(x) at x = -1. Using the fact that f(-1) = -4, write a formula for y = L(x).

*Solution: Note that f(-1) = - 4 and*

*Simplifying: L(x) = -4+3(x+1)*

1. *(6 pts)* Use your formula for L(x) to approximate f(-0.5).

*Solution: Since 0.5 is close to x = -1, our estimate is:*

*f(-0.5) ≈ L(-0.5) = -4 + 3(-0.5+1) =* ***-5.5***

8. Suppose that *f* and *g* are differentiable functions satisfying:

f(3) = -2, g(3) = -4, f ′(3) = 3, and g′(3) = -1.

1. *(6 pts)* Let H(x) = (f(x) + 2g(x) + 1)(f(x) – g(x) – 4). Compute H′(3) (Hint: Use short cuts here.)

*Solution: Using the product rule,*

*(3+(-2)(-2-(-4)-4) + (-2 + (-8) +1)(3-(-1)) = 1(-2) + (-9)(4) =* ***-38***

 Compute M′(3)

*Solution: Using the quotient rule,*

*So,* = =

9. For each of the following, find any and all critical points. Then, using the first derivative test, classify them (local max, local min, neither).

1. *(6 pts)* y = f(x) = x3 – 3x + 1

*Solution:*

*Hence the critical points are x = ±1.*

*Next, performing a sign analysis on dy/dx, we find that dy/dx is positive when |x| > 1 and negative when |x|<1. Hence f is rising on (-∞, -1), falling on (-1, 1), and rising on (1, ∞).*

*This means that f achieves a* ***local maximum*** *at x = -1 and a* ***local minimum*** *at x =1.*

1. *(6 pts)* y = g(x) = 3x4 –16x3 + 18x2 + 1

*Solution:*

*Hence the critical points are x = 0, 1, 3.*

*Next, performing a sign analysis on dy/dx, we find that dy/dx is positive when x>3 and 0<x<1; dy/dx is negative when 1<x<3 and when x<0. . Hence g is rising on (0, 1) and on (3, ∞); g is falling on (-∞, 0) and on (1, 3).*

*This means that g achieves a* ***local maximum*** *at x = 4 and* ***local minima*** *at x = 1 and x = 6.*

10. Let y = f(x) be a differentiable function with derivative

1. *(4 pts)* Find any and all critical points.

 These are the five critical points.

1. *(8 pts)* Classify each critical point (local max, local min, neither).

*Solution: Clearly since their respective factors have even exponents, x = 2 and x =5 are neither max nor min. Doing a sign analysis on*

*Using the first derivative test, we find that x = 4 is a local maximum; x= 1 and x = 6 are both local minima.*

11. *(3 pts each)* Compute each of the following limits. *Justify your reasoning.*



*Solution: Observe that:*





 *Solution: Observe that, as long as x ≠ 3:*



 

*Solution: This limit is 0 due to the squeeze theorem, viz,*

*Since we can assume x > 0,*

*Now 1/x and -1/x →0 as x→ ∞, and so must.*



*Solution: We begin by rationalizing the numerator of the algebraic expression. Then we assume that, as long as x ≠ 0:*



12. *(3 pts each)* For each of the following functions, determine the type of discontinuity at the given point. If it is a *removable* discontinuity, find a continuous extension of the function.

1. at x = 2

*Solution: Factoring yields*

*Now as x →2, y →. Thus this is a* ***removable discontinuity****. The continuous extension requires that y be defined as when x = 2.*

1. at x = -5

*Solution: In part (a), we factored y as follows:*

*Now as x → -5, the numerator tends to 20 but the denominator → 0. Thus this is an* ***infinite discontinuity****.*

1.

*Solution: This is our archetypal example of an* ***essential discontinuity****.*

*Solution: This is a* ***jump discontinuity*** *since*

*It is a hypothesis that the sun will rise tomorrow: and this means that we do not know whether it will rise.*

**- Ludwig Wittgenstein**



