MATH 161 OLD TEST I SEPTEMBER 2017

Instructions: *Answer any 10 of the following 12 questions. You may solve more than 10 to obtain extra credit.* ☺

1. Albertine orders a large cup of coffee at Metropolis on Granville. Let F(t) be the temperature in *degrees Fahrenheit* of her coffee *t minutes* after the coffee is placed on her tray.

(a) Explain the meaning of the statement: F(9) = 167. (Use complete sentences. Avoid any mathematical terms!)

(b) Explain the meaning of the statement: $F^{-1}(99) = 17.5$

(c) Give the *practical* interpretation of the statement: F′(9) = – 1.10. (Use complete sentences. Do not use the words "derivative" or "rate" or any other mathematical term in your explanation.)

(d) What are the *units* of F′(9)?

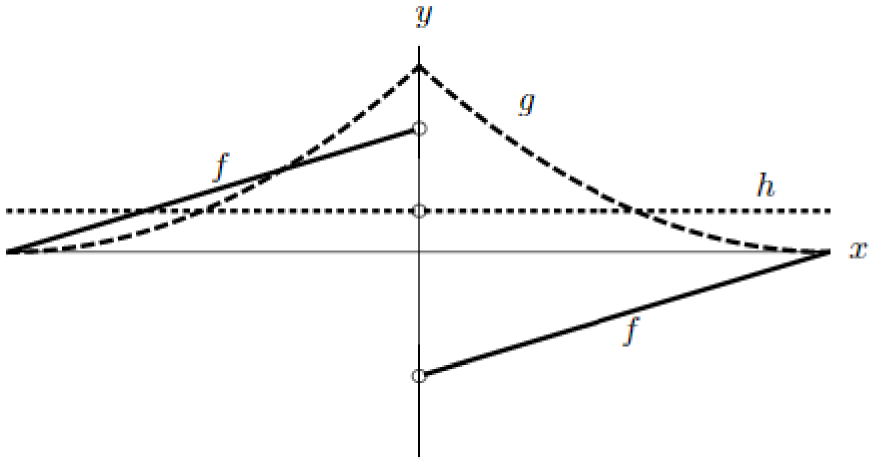
(e) Using the information given in parts (a) and (c), estimate the temperature of Albertine's coffee *seven* minutes after she has been handed the coffee.

(f) [EXTRA CREDIT] Explain the meaning of the statement:

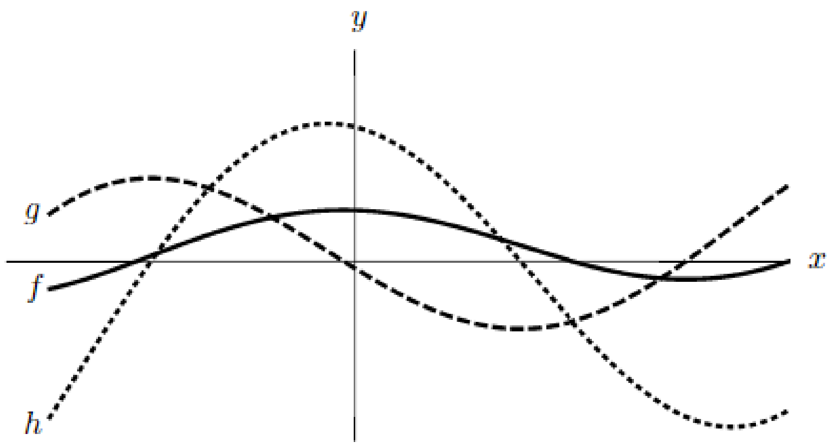
$$(F^{-1})'(99) = -1$$

2. For each of the following three sets of axes, exactly one of the following statements (a) – (e) is true. You may use a letter more than once. In the space provided next to each figure, enter the letter of the true statement for that figure. For each graph, note that the x and y scales are not the same.

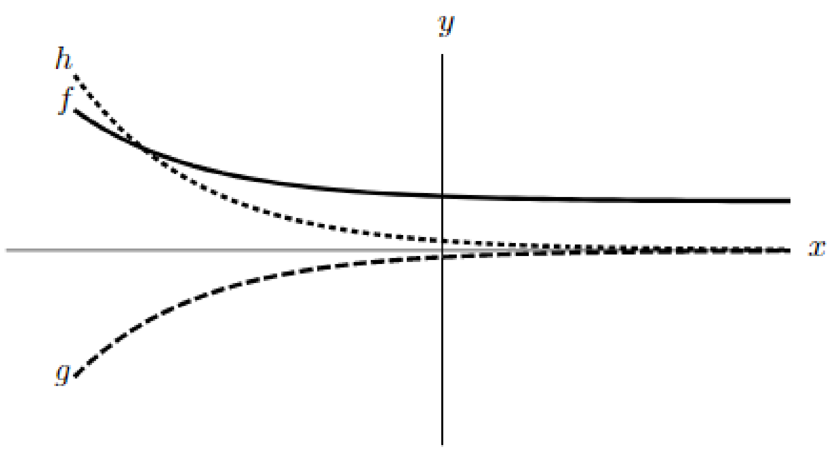
(a) h is the derivative of f, and f is the derivative of g.
(b) g is the derivative of f, and f is the derivative of h.
(c) g is the derivative of h, and h is the derivative of f.
(d) h is the derivative of g, and g is the derivative of f.
(e) None of (a)-(d) are possible.



True statement _____



True statement _____



True statement _____

3. Using the limit definition of the derivative, write an explicit expression for the *derivative* of the function $g(x) = (\cos x)^x$ at $x = 3$. *Do not try to calculate this derivative.*

4. (a) Find $\lim_{x\to\infty} f(x)$ *if, for all* $x > 5$,

$$\frac{4x-1}{x} < f(x) < \frac{4x^2+3x}{x^2}$$

Explain!

Which theorem are you using?

(b) Show that y = f(x) = $x^3 + 5e^x + 1$ has *at least one* real root. *Explain!*

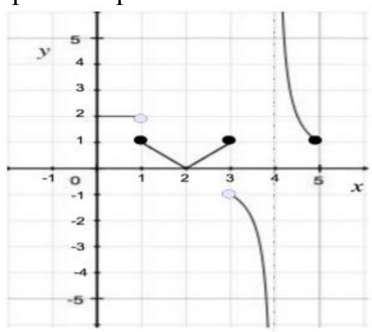
Which theorem are you using?

5. Find an equation of the *normal line* to the curve

$$y = g(x) = \frac{x^2-1}{x^2+1} \quad \text{at } x = 1.$$

You may use short cuts.

6. The graph of a function *g* is shown below. For each of the following, decide if the limit exists. If it does, find the limit. If it does not, decide also if the "limit" is ∞, -∞, or neither. No justification is necessary for full credit, but show your work for purposes of partial credit.



(a) $\lim_{x\to 0^+} g(x) =$

(b) $\lim_{x\to 1^-} g(x)=$

(c) $\lim_{x\to 1^+} g(x)=$

(d) $\lim_{x\to 1} g(x)=$

(e) $\lim_{x\to 2} g(x)=$

(f) $\lim_{x\to 0^+} g(x) =$

(g) $\lim_{x\to 3^-} g(x)=$

(h) $\lim_{x\to 3^+} g(x) =$

(i) $\lim_{x\to 3} g(x) =$

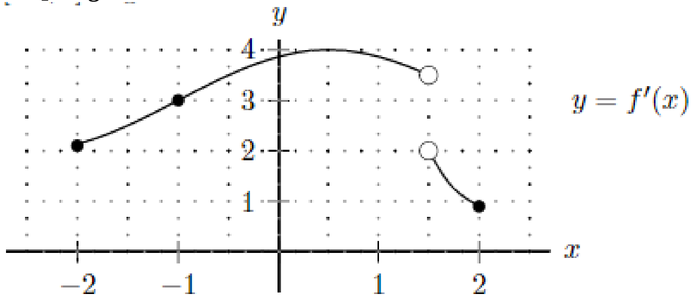
(j) $\lim_{x\to 4^+} g(x) =$

(k) $\lim_{x\to 4^-} g(x) =$

(l) $\lim_{x\to 4} g(x) =$

(m) $\lim_{x\to 5^-} g(x) =$

7. Suppose that f(x) is a functon that is continuous on the interval [-2, 2]. The graph of $f'(x)$ *on the interval* $[-2, 2]$ *is given below.*



(a) Let y = L(x) be the local linerariztion of f(x) at x = -1. Using the fact that f(-1) = -4, write a formula for y = L(x).

(b) Use your formula for L(x) to approximate f(-0.5).

8. Suppose that *f* and *g* are differentiable functions satisfying:

f(3) = -2, g(3) = -4, f ′(3) = 3, and g′(3) = -1.

(a) Let H(x) = (f(x) + 2g(x) + 1)(f(x) – g(x) – 4). Compute H′(3) (Hint: Use short cuts here.)

(b) Let $M(x) = \dfrac{2f(x)+3g(x)}{2-3g(x)}$. Compute M′(3)

9. For each of the following, find any and all critical points. Then, using the first derivative test, classify them (local max, local min, neither).

(a) $y = x^3 - 3x + 1$

(b) $y = 3x^4 - 16x^3 + 18x^2 + 1$

10. Let y = f(x) be a differentiable function with derivative

$$f'(x) = \frac{e^x(x-1)(x-2)^2(x-4)^3(x-5)^4(x-6)^5}{1+x^4}$$

(a) Find any and all critical points.
(b) Classify each critical point (local max, local min, neither).

11. Compute each of the following limits. *Justify your reasoning.*

$$(a)\quad \lim_{x\to\infty} \frac{(4x^3+11)^2(3x-91)^3}{(2x^2+5)^4(2x+2017)}$$

$$(b)\quad \lim_{x\to 3} \frac{\frac{1}{x^2}-\frac{1}{9}}{x-3}$$

$$(c)\quad \lim_{x\to\infty} \frac{\sin 7x}{x}$$

$$(d)\quad \lim_{x\to 0} \frac{\sqrt{x+1}-1}{x}$$

12. For each of the following functions, determine the type of discontinuity at the given point. If it is a *removable* discontinuity, find continuous extension of the function.

(a) $y = \frac{x^3-x^2-2x}{(x-2)(x+5)}$ at x = 2

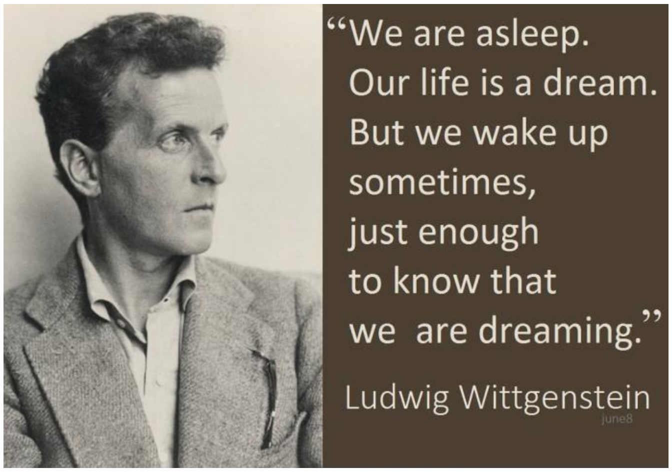
(b) $y = \frac{x^3-x^2-2x}{(x-2)(x+5)}$ at x = -5

(c) $y = cos\frac{3}{x} \quad at\ x = 0$

(d) $y = \frac{|x|}{x} \quad at\ x = 0$

It is a hypothesis that the sun will rise tomorrow: and this means that we do not know whether it will rise.

- Ludwig Wittgenstein



DERIVATIVE RULES

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(a^x\right) = \ln a \cdot a^x$$

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) = f(x)\cdot g'(x) + g(x)\cdot f'(x)$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{\left(g(x)\right)^2}$$

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\cdot g'(x)$$

$$\frac{d}{dx}\left(\sin x\right) = \cos x \qquad \frac{d}{dx}\left(\cos x\right) = -\sin x$$

$$\frac{d}{dx}\left(\tan x\right) = \sec^2 x \qquad \frac{d}{dx}\left(\cot x\right) = -\csc^2 x$$

$$\frac{d}{dx}\left(\sec x\right) = \sec x \tan x \qquad \frac{d}{dx}\left(\csc x\right) = -\csc x \cot x$$

$$\frac{d}{dx}\left(\arcsin x\right) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left(\arctan x\right) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(\operatorname{arcsec} x\right) = \frac{1}{x\sqrt{x^2-1}}$$