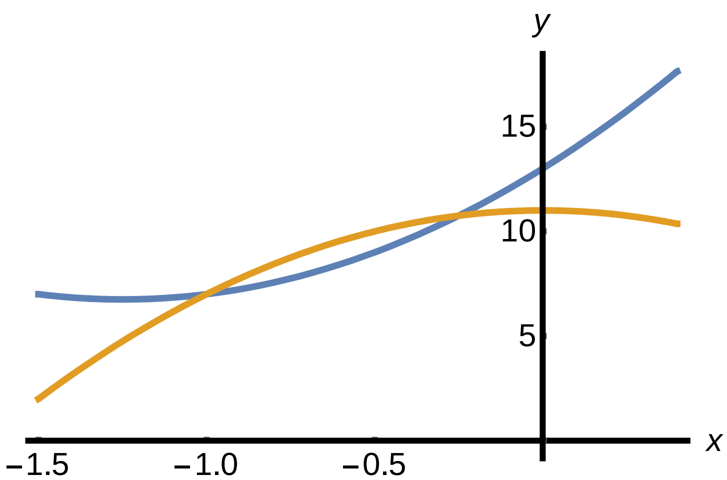
MATH 161 SOLUTIONS: QUIZ X 30 NOVEMBER 2018

1. Find the area between the two curves: $y = 4x^2 + 10x + 13$ and $y = 11 - 4x^2$. *Sketch.*

Solution: Solving for points of intersection, $4x^2 + 10x + 13$ and $y = 11 - 4x^2$



$4x^2 + 10x + 13 = 11 - 4x^2 \Rightarrow 8x^2 + 10x + 2 = 0 \Rightarrow \ 4x^2 + 5x + 1 = 0 \Rightarrow$

$(4x + 1)(x + 1) = 0 \Rightarrow x = -\frac{1}{4}\ , \ x = -1$

Thus the area between the two parabolas is:

$$\int_{-1}^{-\frac{1}{4}} \left((11 - 4x^2) - (4x^2 + 10x + 13)\right) dx = \frac{9}{16} \approx 0.563$$

2. Let $F(x) = \sin x + \int_{-3}^{\cos x} \frac{1}{5+t^3} dt$. *Find* $F'(x)$ . .

Solution: Applying the FTC, $F'(x) = \cos x + \frac{-\sin x}{5+\cos^3 x} = \mathbf{\cos x} - \frac{\mathbf{\sin x}}{\mathbf{5+\cos^3 x}}$

3. Show, using only algebra, that

$$\cosh(2x) = (\cosh x)^2 .+ (\sinh x)^2$$

Solution: $(\cosh x)^2 .+ (\sinh x)^2 =$

$$\frac{1}{4}\left(e^{x}+e^{-x}\right)^{2}+\frac{1}{4}\left(e^{x}-e^{-x}\right)^{2}=$$

$$\frac{1}{4}\left(e^{2x}+2+e^{-2x}+e^{2x}-2+e^{-2x}=\right.$$

$$\frac{1}{4}\left(2e^{2x}+2e^{-2x}\right)=$$

$$cosh(2x)$$

4. Find the area beneath a curve, using the FTC (main version). Find the area beneath the curve $f(x)=\frac{x^{6}}{5+3x^{7}}$ that is above the interval [0, 1].

Solution: The area of this region is $\int_0^1 \frac{x^6}{5+3x^7}\,dt$

Applying the FTC:

$$\int_0^1 \frac{x^6}{5+3x^7}\,dt=\frac{1}{21}\ln(5+3x^7)\,\Big|_0^1 \;=\frac{1}{21}\,(\ln 8-\ln 5)=\boldsymbol{\frac{1}{21}\ln\frac{8}{5}}$$

5. Let C(t) be the temperature, in degrees Fahrenheit, of a warm can of soda t minutes after it was put in a refrigerator. Suppose C(10) = 62.

(a) Assuming C has an inverse, give a practical interpretation of the statement $C^{-1}(45)=40$

Solution: It was 40 minutes after the soda was put into the refrigerator when the temperature of the soda was 45 degrees Fahrenheit. (Or the temperature of the soda was 45 degrees Fahrenheit after 40 minutes in the refrigerator.)

(b) Give a practical interpretation of the statement. $C'(10)=-0.4$

Solution: After 10 minutes in the refrigerator, the temperature of the soda would decrease by about 0.4 degrees Fahrenheit during the next minute.

(c) Give a practical interpretation of the statement $\int_0^{10} C'(t)dt=-5$

Solution: The temperature of the can of soda decreased by 5 degrees Fahrenheit during the first 10 minutes it was in the refrigerator.

(d) Assuming the statements in parts (a)-(c) are true, determine C(0).

Solution: By the Fundamental Theorem of Calculus, we have
$C(10) - C(0) = \int_0^{10} C'(t)dt \ = -5$
Since it is given that C(0) = 62, we have C(10) = 62 – (-5) = ***67° F****.*

(e) What is the practical meaning of:

$$\int_0^1 C(t)dt?$$

Solution:

$$\int_0^1 C(t)dt = \frac{1}{1-0}\int_0^1 C(t)dt$$

Thus the integral represents the average temperature, in degrees Fahrenheit, of the can of soda during the first minute in the refrigerator.

Extra Credit: Suppose $\int_0^{2x+1} f(t)\ dt = x\sqrt{5x+9}$

Find $f(1)$

Solution: Applying the FTC, $\frac{d}{dx}\ \int_0^{2x+1} f(t)\ dt = 2f(2x+1)$

$$\frac{d}{dx}\ x\sqrt{5x+9} = 1\sqrt{5x+9} + x\frac{1}{2}\ (5x+9)^{-\frac{1}{2}}5 =$$

$$\sqrt{5x+9} + \frac{5}{2}\ x(5x+9)^{-\frac{1}{2}}$$

Hence $2f(2x+1) = \sqrt{5x+9} + \frac{5}{2}\ x(5x+9)^{-\frac{1}{2}}$

Setting x = 0, we have $2f(1) = \sqrt{0+9} + \frac{5}{2}\ 0(0+9)^{-\frac{1}{2}}\ = 3$

Hence $\boldsymbol{f(1) = \frac{3}{2}}$