MATH 161 SOLUTIONS: QUIZ II

Compute each of the following limits or show why the limit fails to exist. Circle each answer. *Justify your reasoning.*

(a) [6 pts] $\lim_{x\to 1} \frac{(x-5)^5}{x^{2019}+4x-6}$

Solution:

$$\lim_{x\to 1} \frac{(x-5)^5}{x^{2019}+4x-6} = \frac{\lim_{x\to 1}(x-5)^5}{\lim_{x\to 1}(x^{2019}+4x-6)} = \frac{(-4)^5}{1+4-6} = -\frac{2^{10}}{-1} = 2^{10}$$
$$= 1024$$

(b) [6 pts] $\lim_{x\to 1} \frac{6x^2+x-2}{2x-1}$

Solution:

$$\lim_{x\to 1} \frac{6x^2+x-2}{2x-1} = \frac{\lim_{x\to 1}(6x^2+x-2)}{\lim_{x\to 1}(2x-1)} = \frac{6+1-2}{2-1} = 5$$

(c) [6 pts] $\lim_{x\to \infty} \frac{\sin^4 x+\cos^3 x+\sqrt{x}}{x+1}$

Solution: Since $-1 \le \sin^4 x + \cos^3 x \le 2$, we observe that $\sqrt{x}$ is the dominant term of the numerator. Thus

$$\frac{\sin^4 x+\cos^3 x+\sqrt{x}}{x+1} \cong \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}} \to 0 \;\; as \;\; x \to \infty$$

Hence $\lim_{x\to\infty} \frac{\sin^4 x+\cos^3 x+\sqrt{x}}{x+1} = 0$

(d) [6 pts] $\lim_{x\to\infty} \frac{x(x-1)^7(3x-1)^2}{(x^2+1)^3(2x+2019)^4}$

Solution: Since each of the numerator and denominator increase without bound as $x \to \infty$, we choose the dominant term from every factor:

$$\frac{x(x-1)^7(3x-1)^2}{(x^2+1)^3(2x+2019)^4} \cong \frac{x\,x^7(3x)^2}{(x^2)^3(2x)^4} = \frac{9x^{10}}{16x^{10}} = \frac{9}{16}$$

Hence $\lim_{x\to\infty} \frac{x(x-1)^7(3x-1)^2}{(x^2+1)^3(2x+2019)^4} = \frac{9}{16}$

(e) [6 pts] $\lim_{x\to\infty} \frac{1+512e^{5x}}{x^2+2028e^x\,(1+e^{2x})^2}$

Solution: Since each of the numerator and denominator increase without bound as $x \to \infty$, we choose the dominant term from every factor.

$$\frac{1+512e^{5x}}{x^2+2028e^x\,(1+e^{2x})^2} \cong \frac{512e^{5x}}{x^2+2028e^x\,(e^{2x})^2} =$$

$$\frac{512e^{5x}}{x^2+2028e^{5x}} \cong \frac{512e^{5x}}{2028e^{5x}} = \frac{512}{2028} = \frac{128}{507}$$

(f) [6 pts] $\lim_{x\to 5} \frac{1-\frac{5}{x}}{3-\frac{16}{x}+\frac{5}{x^2}}$

Solution: $As\ x \to 5, 1 - \frac{5}{x} \to 0\ and\ 3 - \frac{16}{x} + \frac{5}{x^2} \to 0.$
Thus, we are dealing with an indeterminate form of the type $\frac{0}{0}$. So, using algebra:

$$\frac{1-\frac{5}{x}}{3-\frac{16}{x}+\frac{5}{x^2}} = \frac{1-\frac{5}{x}}{3-\frac{16}{x}+\frac{5}{x^2}} \cdot \frac{x^2}{x^2} = \frac{x^2-5x}{3x^2-16x+5} =$$

$$\frac{x(x-5)}{(x-5)(3x-1)} = \frac{x}{3x-1} \quad provided\ x \neq 5$$

Now $\frac{x}{3x-1} \to \frac{5}{14}\ as\ x \to 5.\ \ Thus$

$$\lim_{x\to 5} \frac{1-\frac{5}{x}}{3-\frac{16}{x}+\frac{5}{x^2}} = \frac{5}{14}$$

(g) [6 *pts*] $\lim_{x\to 1}\left(1+\frac{4}{x+1}+\sin \pi x - 3\cos \pi x\right)$

Solution: Using the limit law that states the limit of a sum equals the sum of the limits of the summands provided the limit of each summand exists, we obtain:

$$\lim_{x\to 1}\left(1+\frac{4}{x+1}+\sin \pi x - 3\cos \pi x\right)$$
$$= \lim_{x\to 1} 1 + \lim_{x\to 1}\frac{4}{x+1} + \lim_{x\to 1}\ \sin \pi x + \lim_{x\to 1}\ (-3\cos \pi x)$$
$$= 1 + \frac{4}{1+1} + 0 + 3 = 6$$

Extra Credit (**Nipissing University**, ON, Canada)

1. [6 pts] Evaluate the follosing limit, if it exists:

$$\lim_{x\to 4} \frac{2x^3 - 128}{\sqrt{x} - 2}$$

Solution:

$$\frac{2x^3 - 128}{\sqrt{x} - 2} = 2 \ \frac{x^3 - 4^3}{\sqrt{x} - 2} = 2 \ \frac{(x-4)(x^2 + 4x + 16)}{\sqrt{x} - 2}$$

$$= 2 \ \frac{(x-4)(x^2 + 4x + 16)}{\sqrt{x} - 2} =$$

$$= 2 \ \frac{\left(\sqrt{x} - 2\right)\left(\sqrt{x} + 2\right)(x^2 + 4x + 16)}{\sqrt{x} - 2}$$

$$= \ 2 \ \left(\sqrt{x} + 2\right)(x^2 + 4x + 16) \ provided \ x \neq 4.$$

Thus

$$\lim_{x\to 4} \frac{2x^3 - 128}{\sqrt{x} - 2} = \lim_{x\to 4} 2 \ \left(\sqrt{x} + 2\right)(x^2 + 4x + 16)$$

$$= 2 \ \left(\sqrt{4} + 2\right)(4^2 + 4 \cdot 4 + 16) = 384$$

2. [6 pts] Find the value k for which the following limit exists:

$$\lim_{x\to 3} \frac{4x^2 + kx + 7k - 6}{2x^2 - 5x - 3}$$

Solution:

First observe that

$$\lim_{x\to 3}\ (2x^2 - 5x - 3) = 2(9) - 5(3) - 3 = 0$$

Now, if the limit has a value, then we must be looking at an indeterminate form. In other words, the numerator must also tend toward 0. So:

$$0 = \lim_{x\to 3}(4x^2 + kx + 7k - 6) = 36 + 3k + 7k - 6 = 30 + 10k$$

Thus k = -3.

The limits of my language are the limits of my world.

- Ludwig Josef Johann Wittgenstein, **Tractatus Logico-Philosophicus**