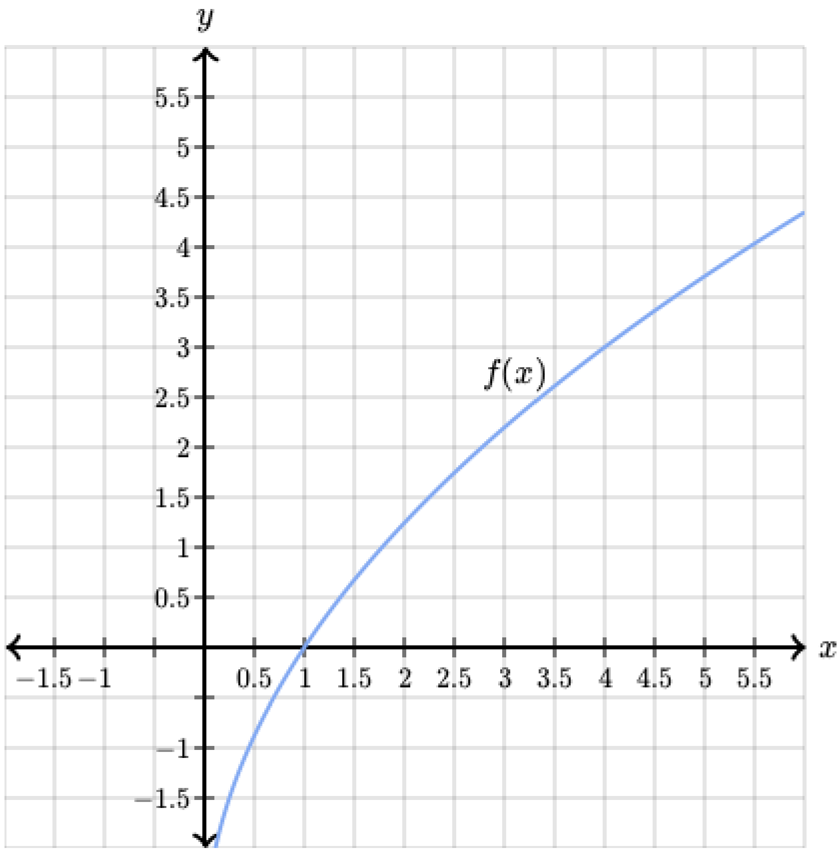
MATH 161 SOLUTIONS: QUIZ V 5 OCTOBER 2018

1. *[4 pts]*

Which of the following comparisons of $f(1)$, $f'(1)$, and $f''(1)$ is true?



Choose 1 answer:

(A) $f(1) < f'(1) < f''(1)$

(B) $f(1) < f''(1) < f'(1)$

(C) $f'(1) < f(1) < f''(1)$

(D) $f''(1) < f(1) < f'(1)$

Solution: ***D*** *is the correct answer. Note that* $f''(1) < 0$

Since f is concave down over an interval that includes x = 1.

Next, f(1) = 0 which we can see from the graph.

Finally $f'(1) > 0$ *since f is increasing on an interval containing x = 1.*

2. *Given a function y = f(x) for which* $\frac{dy}{dx} = x^3(x-4)$

(a) [2 pts] *Find the critical points of f(x)?*

Solution: We are given that $f'(x) = x^3(x-4)$. *Setting* $f'(x) = 0$, *we find x = 0, x =4 are critical points.*

(b) [2 pts] *Determine the interval(s) upon which f(x) is rising.*

Solution: f(x) is rising when $f'(x) > 0$. *Now the sign analyisis of* $f'(x)$ *has two transition points, x = 0 and x = 4.*

We see that $f'(x) > 0$ *when* $x < 0$ *or when* $x > 4$.

(c) [3 pts] *Determine the interval(s) upon which f(x) is concave up.*

Solution: Using the product rule, $f''(x) = x^3 \frac{d}{dx}(x-4) + (x-4)\frac{d}{dx} x^3 = x^3 + (x-4)3x^2 =$

$x^2(x + 3(x-4)) = x^2(4x-12) = 4x^2(x-3)$.

In performing a sign analysis on $f''(x)$ *we have only one transition point, viz. x =3 .*

Hence f is concave up on $(\mathbf{3}, \infty)$.

3. Find an *anti-derivative* for each of the following functions. The method of *judicious* guessing is highly recommended. *Show your work.*

(a) *[2 pts]* $e^x - 3x^8$

Answer: $e^x - \frac{1}{3}x^9$

(b) *[2 pts]* $\sec^2 x + 5 \sec x \tan x + \pi^9$

Answer: $\tan x + 5\sec x + \pi^9 x$

Notice that π^9 *is a constant!*

(c) *[2 pts]* $1 + 3x^2 - 18x^5$

Answer: $x + x^3 - 3x^6$

(d) *[2 pts]* $1 + 3e^x + 4\cos x$

Answer: $x + 3e^x + 4\sin x$

(e) *[3 pts]* $(3+x^3)^2$

Solution: Using algebra: $(3+x^3)^2 = 9 + 6x^3 + x^6$. *Hence an antiderivative of* $(3+x^3)^2$ *is*

$$9x + \frac{3}{2}x^3 + \frac{1}{7}x^7$$

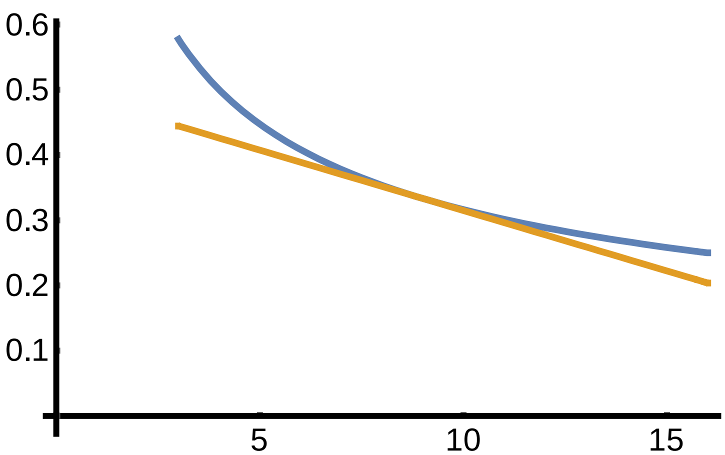
(f) *[3 pts]* $\frac{1+x+x^3}{x^3}$

Solution: Using algebra: $\frac{1+x+x^3}{x^3} = x^{-3} + x^{-2} + 1$. *Hence an antiderivative of* $\frac{1+x+x^3}{x^3}$ *is*

$$-\frac{1}{2}x^{-2} - x^{-1} + x$$

4. *[7 pts]* (a) Using an appropriate function and an appropriate point, estimate $\frac{1}{\sqrt{8.994}}$. Sketch!

Solution: Let $f(x) = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$ *and let the point be* $x = 9$.



Since $f'(x) = -\frac{1}{2}x^{-\frac{3}{2}}$, $f'(9) = -\frac{1}{54}$

So the linearization of f(x) at x = 9 is given by $L(x) = \frac{1}{3} - \frac{1}{54}(x-9)$.

Thus our approximation of $\frac{1}{\sqrt{8.994}}$ *is* $L(8.994) = \frac{1}{3} - \frac{1}{54}(8.994-9) = \frac{1}{3} - \frac{1}{54}(-0.006) = \frac{1}{3} + \frac{0.006}{54} \approx$

0.33344444444.

[3 pts] (b) Is this an over or an underestimate? Why? (No credit for calculator answers.)

Solution: This is an ***underestimate*** *since the tangent line to y = f(x) at x = 9 lies below the curve. Cf. sketch above. Equivalently f(x) is concave up at x = 9 because* $f''(9) > 0$.

DERIVATIVE RULES

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(\sin x\right) = \cos x$$

$$\frac{d}{dx}\left(\cos x\right) = -\sin x$$

$$\frac{d}{dx}\left(a^x\right) = \ln a \cdot a^x$$

$$\frac{d}{dx}\left(\tan x\right) = \sec^2 x$$

$$\frac{d}{dx}\left(\cot x\right) = -\csc^2 x$$

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) = f(x)\cdot g'(x) + g(x)\cdot f'(x)$$

$$\frac{d}{dx}\left(\sec x\right) = \sec x \tan x$$

$$\frac{d}{dx}\left(\csc x\right) = -\csc x \cot x$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{\left(g(x)\right)^2}$$

$$\frac{d}{dx}\left(\arcsin x\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\arctan x\right) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\cdot g'(x)$$

$$\frac{d}{dx}\left(\operatorname{arc sec} x\right) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(\ln x\right) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\sinh x\right) = \cosh x$$

$$\frac{d}{dx}\left(\cosh x\right) = \sinh x$$