MATH 161 SOLUTIONS: QUIZ VI 12 OCTOBER 2018

1. Find an anti-derivative of each of the following functions:

(a) $\frac{2+x^7}{x^9}$

Solution: $\frac{2+x^7}{x^9} = 2x^{-9} + x^{-2}$

A good first guess might be $x^{-8} + x^{-1}$

Correcting for constants the answer is $-\frac{1}{4}x^{-8} - x^{-1}$

(b) $3e^x + 4\sec^2 x + \pi$

Solution: A good first guess might be $e^x + \tan x + \pi x$.

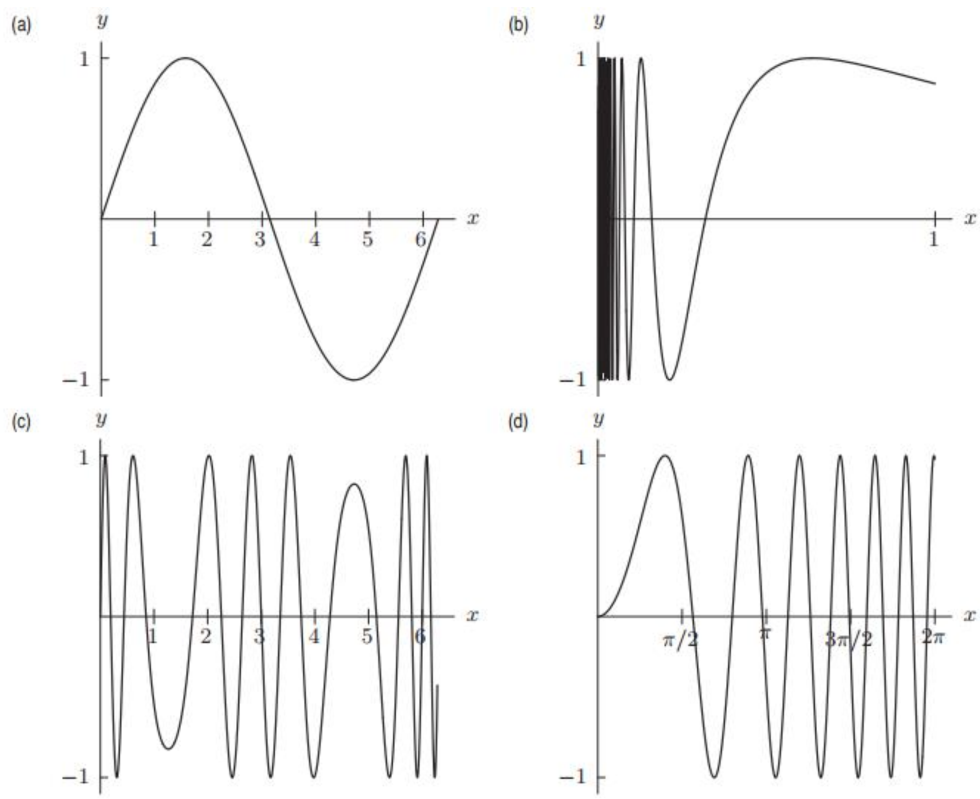
Correcting for constants the answer is $\mathbf{3e^x + 4\tan x + \pi x}$

(c) $\cos x \tan x$

Solution: First note that $\cos x \tan x = \cos x \frac{\sin x}{\cos x} = \sin x$

Thus an antiderivative that we seek is $-\cos x$

2. *[1 pt]* Which of the graphs below is that of $\sin(x^2)$?



Answer: **D**

(d). Since $\frac{d}{dx}\sin(x^2) = 2x\cos(x^2)$, the maximum value of the slope of the graph increases as x increases. Also, the zeros of $\sin(x^2)$ are $x = \sqrt{n\pi}$ for $n = 0, 1, 2, 3, \ldots$, which are closer together as x increases.

3. *[1 pt]* Given the graphs of the functions f(x) and g(x) in figures 3.7 and 3.8, which of the following (a) – (d) is a graph of $f \circ g(x)$?

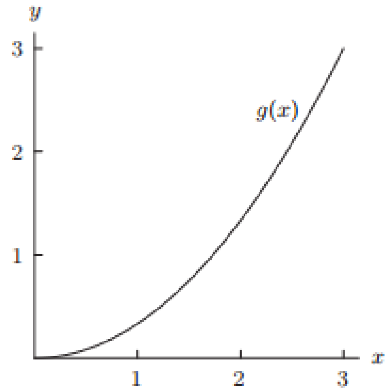


Figure 3.7

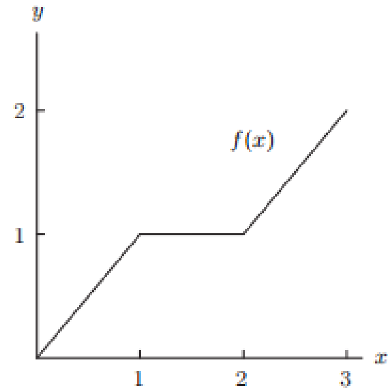
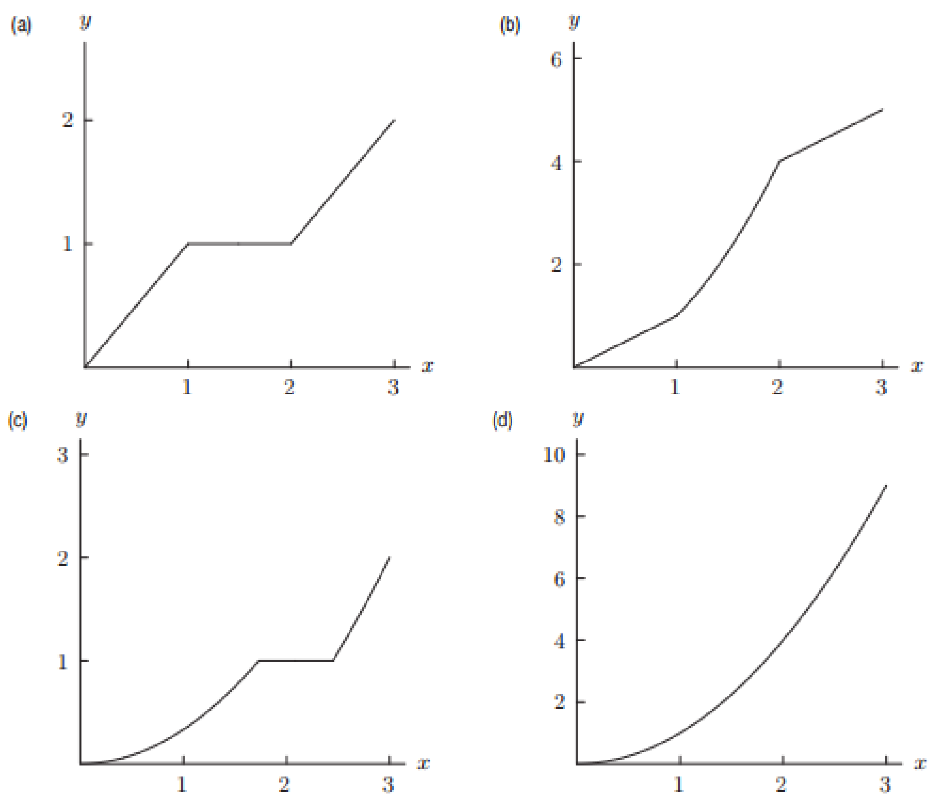


Figure 3.8



Answer: ***C***

Reason: Because (f(g(x)))′ = f ′ (g(x))g ′ (x), we see f(g(x)) has a horizontal tangent whenever g′(x) = 0 or f ′(g(x)) = 0. Now, f ′(g(x)) = 0 for 1 < g(x) < 2 and this approximately corresponds to 1.7 < x <2.5.

4. Using the chain rule compute the derivative of each of the following functions. You need not simplify.

(a) e^{1+x^3}

Solution: Using the chain rule (shortcut):

$$\frac{d}{dx}e^{1+x^3} = e^{1+x^3}\frac{d}{dx}(1+x^3) = \mathbf{3x^2e^{1+x^3}}$$

(b) $\tan(\sin x)$

Solution:

Using the chain rule (shortcut):

$$\frac{d}{dx}\tan(\sin x) = sec^2(\sin x)\ \frac{d}{dx}(\sin x) = \mathbf{(\cos x)\ sec^2(\sin x)}$$

(c) $(1+\mathrm{x}+x^3)^{2019}$

Solution: Using the general power rule:

$$\frac{d}{dx}(1+\mathrm{x}+x^3)^{2019} = 2019(1+\mathrm{x}+x^3)^{2018}\frac{d}{dx}(1+\mathrm{x}+x^3) =$$

$$2019(1+\mathrm{x}+x^3)^{2018}(1+3x^2) = \mathbf{2019(1+3x^2)(1+x+x^3)^{2018}}$$

(d) $e^{5x}\cos(3x+1)$

Solution: First using the product rule:

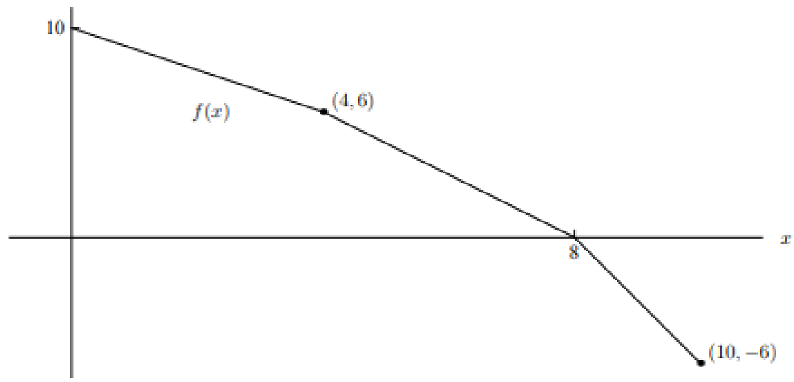
$$\frac{d}{dx}e^{5x}\cos(3x+1) = e^{5x}\frac{d}{dx}\cos(3x+1) + \cos(3x+1)\frac{d}{dx}e^{5x} =$$

$$e^{5x}\left(-\sin(3x+1)\frac{d}{dx}(3x+1)\right) + \cos(3x+1)e^{5x}\frac{d}{dx}(5x) =$$

$$\mathbf{-3e^{5x}\sin(3x+1) + 5\,e^{5x}\cos(3x+1)}$$

Extra Credit:

Consider the piecewise linear function f(x) graphed below:



For each function g(x) find the value of $g'(3)$.

(*a*) $g(x) = \sin\left(\left(f(x)^3\right)\right)$

Solution: Using the chain rule:

$$g(x) = \frac{d}{dx}\sin\left((f(x)^3)\right) = \cos\left((f(x)^3)\right)\frac{d}{dx}(f(x)^3) =$$

$$\cos\left((f(x)^3)\right)\frac{d}{dx}(f(x)^3) = \cos\left((f(x)^3)\right)3(\mathrm{f(x)})^2 f'(x)$$

Now: Since the slope of the curve at x=3 is $\frac{6-0}{4-10} = -1$, the equation of the curve in the vicinity of x = 3 is

f(x) = -1(x) + 10.

So f(3) = 7 $and\ f'(3) = -1.$

$Hence\ g'(3) = \cos\left((f(3)^3)\right)3(\mathrm{f(3)})^2 f'(3) =$

$$\cos(7^3)3(7)^2(-1) = -147\cos(7^3) \cong 124.04$$

(*b*) $g(x) = \frac{f(x^2)}{x}$

Solution: Using the quotient rule,

$$g'(x) = \frac{x\frac{d}{dx}f(x^2) - f(x^2)\frac{d}{dx}(x)}{x^2}$$

Invoking the chain rule to compute $\frac{d}{dx}f(x^2), we\ find\ that$

$$g'(x) = \frac{x\left(f'(x^2)\right)2x - f(x^2)}{x^2}$$

And so

$$g'(3) = \frac{3(f'(9))2(3) - f(9)}{9}$$

Since f(9) = -3 and $f'(9) = -3, we\ have$:

$$g'(3) = \frac{3(-3)2(3) - (-3)}{9} = -\frac{17}{3} \cong -5.67$$

DERIVATIVE RULES

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(\sin x\right) = \cos x$$

$$\frac{d}{dx}\left(\cos x\right) = -\sin x$$

$$\frac{d}{dx}\left(a^x\right) = \ln a \cdot a^x$$

$$\frac{d}{dx}\left(\tan x\right) = \sec^2 x$$

$$\frac{d}{dx}\left(\cot x\right) = -\csc^2 x$$

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) = f(x)\cdot g'(x) + g(x)\cdot f'(x)$$

$$\frac{d}{dx}\left(\sec x\right) = \sec x \tan x$$

$$\frac{d}{dx}\left(\csc x\right) = -\csc x \cot x$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{\left(g(x)\right)^2}$$

$$\frac{d}{dx}\left(\arcsin x\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\arctan x\right) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\cdot g'(x)$$

$$\frac{d}{dx}\left(\operatorname{arc}\sec x\right) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(\ln x\right) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\sinh x\right) = \cosh x$$

$$\frac{d}{dx}\left(\cosh x\right) = \sinh x$$

O dear Ophelia!
I am ill at these numbers:
I have not art to reckon my groans.

- HAMLET (Act II, Sc. 2)