

**Instructions:** Answer any 3 of the following 4 problems. You may answer all 4 to earn extra credit.

1. Suppose that the side of a huge cube of ice is *decreasing* at a rate of 0.5 cm/min when the side has length 13 cm. At what rate is the *surface area* of the cube changing at that moment?



**Solution:**

Let  $x$  = side of cube (in cm). Let  $S$  = surface area of the cube (in  $\text{cm}^2$ ).

Given:  $dx/dt = -0.5$  when  $x = 13$ .

Find:  $dS/dt$  when  $r = 8$

Since  $S = f(x)$  and  $x = g(t)$ , we can invoke the Chain Rule:

$$\frac{dS}{dt} = \frac{dS}{dx} \frac{dx}{dt}$$

Since a cube has 6 faces,  $S = 6x^2$ .

Then  $dS/dx = 12x$ .

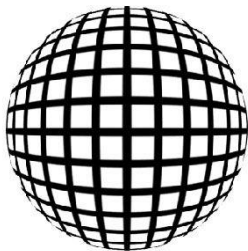
Substituting in the Chain Rule above:

$$\frac{dS}{dt} = (12x)(-0.5)$$

When  $x = 13$ ,

$$\frac{dS}{dt} = 12(13)(-0.5) = -78 \text{ cm}^2/\text{min}$$

2. Suppose that a balloon (modeled as a sphere) of radius  $r$  is being inflated and that, at the



moment when  $r = 8$  cm, its radius is changing at the rate of 3 cm/sec. How quickly is the *surface area* of the balloon changing at the moment when the balloon's radius is 8 cm? (Hint: The surface area of a sphere of radius  $r$  is given by  $S = 4\pi r^2$ .)

**Solution:**

Let  $r$  denote the radius of the sphere (in cm).

Let  $S$  be the surface area of the sphere (in  $\text{cm}^2$ ).

Given:  $dr/dt = 3$  when  $r = 8$ .

Find:  $dS/dt$  when  $r = 8$

Since  $S = f(r)$  and  $r = g(t)$ , we can invoke the Chain Rule:

$$\frac{dS}{dt} = \frac{dS}{dr} \frac{dr}{dt}$$

Now:  $dS/dr = 8\pi r$ ; evaluating at  $r = 8$ :  $dS/dr = 64\pi$ .

Since  $dr/dt = 3$ , we have:

$$dS/dt = (64\pi) 3 = \mathbf{192\pi \text{ cm}^2/\text{sec}} \approx \mathbf{603.2 \text{ cm}^2/\text{sec}}$$

3. Harry, the potter, has a fixed volume of clay ( $28.3 \text{ cm}^3$ ) in the form of a cylinder. As he rolls the clay, the length of the cylinder,  $L$ , increases, while the radius,  $r$ , decreases. If the length of the cylinder is increasing at a constant rate of  $0.2 \text{ cm}$  per second, find the rate at which the radius is changing when the radius is  $1.5 \text{ cm}$ , and the length is  $4 \text{ cm}$ . (You need not simplify your numerical answer.)

**Solution:**

We are given that  $dV/dt = 0$  (since the potter has a fixed volume of clay).

Also, we are given that  $dh/dt = 0.2$ .

We know that  $V = \pi r^2 h$ . Since each of  $r$  and  $h$  is a function of time, we may differentiate implicitly to obtain:

$$0 = dV/dt = \pi \{ r^2 (dh/dt) + 2rh (dr/dt) \}.$$

Thus  $0 = 0.2 r^2 + 2rh (dr/dt)$ .



When  $r = 1.5$  and  $h = 4$ , we find that:

$$0 = 0.2(1.5)^2 + 2(1.5)(4) (dr/dt)$$

Hence  $dr/dt = -1.5/8 = \mathbf{-0.0375 \text{ cm/sec}}$

4. Albertine and Jean-Luc were friends in high school but then went to college in different parts of the country. They thought they were going to see each other in Detroit during December break, but their schedules didn't match up. It turns out that Albertine is leaving on the same day that Jean-Luc is arriving.

Shortly before Jean-Luc's train arrives in Detroit, he sends a text to Albertine to see where she is, and Albertine sends a text response to say that, sadly, her train has already left. At the moment Albertine sends her text, she is 20 miles due east of the center of the train station and moving east at 30 mph while Jean-Luc is 10 miles due south of the train station and moving north at 50 mph.

- (a) What is the distance between Jean-Luc and Albertine at the time Albertine sends her text? *Remember to include units.*



**Solution:**

*Let  $x$ ,  $y$ ,  $z$  be the distances (in miles) between Albertine and the station, Jean-Luc and the station, and Albertine and Jean-Luc, respectively.*

*Then, since the triangle is a right triangle,  $x^2 + y^2 = z^2$ . When Albertine sends her text,*

*$x = 20$  and  $y = 10$ . So Albertine and Jean-Luc are  $\sqrt{20^2 + 10^2} = 10\sqrt{5} \approx$*

**22.4 miles**

- (b) When Albertine sends her text, are she and Jean-Luc moving closer together or farther apart? How quickly? (*You need not simplify your numerical answer.*) *You must show your work clearly to earn any credit. Remember to include units.*

**Solution:**

*Differentiating  $x^2 + y^2 = z^2$  implicitly (wrt  $t$ ) yields:*

$$2x(dx/dt) + 2y(dy/dt) = 2z(dz/dt)$$

*When Albertine sends her text, we know that  $dx/dt = 30$  and  $dy/dt = -50$ . Hence*

$$\frac{dz}{dt} = \frac{x \frac{dx}{dt} + y \frac{dy}{dt}}{z} = \frac{(20)(30) + (10)(-50)}{\sqrt{500}} = 2\sqrt{5} \approx 4.47$$

*Since, at this time, the sign of  $dz/dt$  is positive, Albertine and Jean-Luc are moving farther apart at a rate of approximately **4.47 mph**.*

*The only way to learn mathematics is to do mathematics. That tenet is the foundation of the do-it-yourself, Socratic, or Texas method.*