# MATH 161 Solutions: Quiz VIII 2 November 2018

***1.*** *[14 pts]* Let g(x) = 3x5 – 5x3 be defined on the real line.

1. Find all the *critical points* of *g*.

***Solution:*** Since , the critical points of g are x = 0, 1, -1

1. Where is *g* *rising*? (Give the appropriate intervals.)

***Solution:*** Performing a sign analysis on we discover that g rises on

1. Find and classify all local extrema. Justify your answer. (Do not say “because the calculator tells me.”)

***Solution:*** Using the first-derivative test, we discover that *g* has a local maximum at x = -1 and a local minimum at x = 1.

1. Does g achieve a global max or global min? Explain.

***Solution:*** Neither a global max nor a global min. In the long run, g(x) behaves as x5 which is unbounded above as well as unbounded below.

1. Where is *g* *concave up*? (Give the appropriate intervals.) Find any and all *points of inflection*.

***Solution:*** .

***Performing a sign analysis on***  , we find three points of inflection:

***Moreover, g is concave up on*** *.*

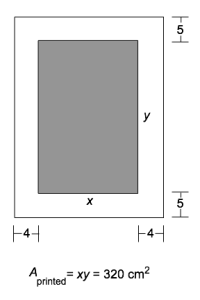
1. Sketch a graph of y = g(x). Label **all local and global extrema** and **all inflection points**. Show regions of increase and decrease. Show regions where the function is concave up and concave down.



2. *[8 pts]* Albertine is designing a rectangular poster to contain 50 in2 of printing with a 4-inch margin at the top and bottom and a 2-inch margin at each side. Which overall dimensions *minimize* the amount of poster board used?

***Solution:*** Let x = width of the printed region (in inches) and let y = length of printed region (in inches).

Hence the outer width is x + 4 inches and the outer length is y + 8 inches.

We are asked to minimize A = (x + 4)(y + 8).

We area given that xy = 50, from which we see that

Thus .

The appropriate domain of A is

Next

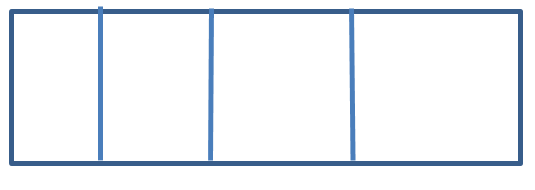
So the critical points are . Of course, we reject , so the unique critical point in our domain occurs at x =

And .

The first derivative test verifies that this is a local minimum, and hence, in our domain, a global minimum.

Hence the overall dimensions are:

**9 inches by 18 inches.**

3. *[8 pts]* Build a rectangular pen with three parallel partitions using 500 feet of fencing. What dimensions will *maximize* the total area of the pen? 

***Solution:*** Let x = length (in ft) of the pen and let y = width (in ft) .

Then the total amount of fencing is 500 = 2x + 5y.

We wish to maximize the area of the pen: A = xy.

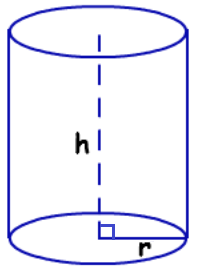
Now . The constraints on x are: .

So Next,

This concave-down parabola has a maximum at **x = 125 feet. And at x = 125, y = 50 feet.**



4. *[8 pts]* A container in the shape of a right circular cylinder with no top has surface area ft2. What height *h* and base radius *r* will maximize the volume of the cylinder?



***Solution:*** We are given that , or equivalently, , from which

The constraints on *r* are

We are asked to maximize subject to the above constraints.

Now . Now the unique critical point in our domain is r = 1.

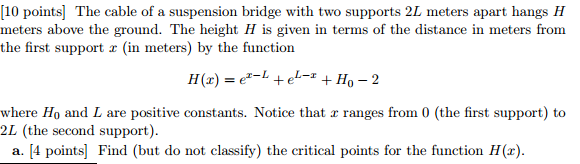
We can easily confirm, using the first derivative test, that r = 1 is a global max in our domain. Now, when r = 1, h = 1.

So the volume of the cylinder is maximized when **r = 1 ft and h = 1 ft.**

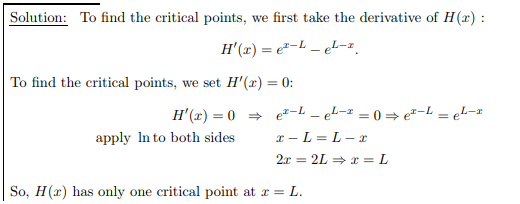


Extra Credit *[University of Michigan problem]*

The cable of a suspension bridge with two supports *2L* meters apart hangs *H*



1. Find (but do not classify) the critical points for the function *H(x).*



1. Find the *x* and *y* coordinates of all global maxima and minima for the function *H(x)*. Justify your answers!

