MATH 162 PRACTICE FINAL EXAMINATION B

Answer any 30 of the following 35 questions. You may answer more than 30 to obtain extra credit. In general, show your work.

1. Give the form of the partial fraction decomposition of $\frac{1}{x^3(x^2+13)^2}$. *Do not* solve for constants A, B, C, etc.

2. Evaluate the improper integral $\int_2^{\infty} \frac{1}{x(\ln x)^3} dx$.

3. Evaluate $\int \arccos x \, dx$.

4. Consider the sequence

$$a_n = \frac{\ln n}{\ln 2n},$$

defined for $n \geq 1$. Does this sequence converge or diverge? In the case of convergence, find its limit.

5. Find the equation of a curve in the xy-plane that passes through the origin and whose arc length from x = 0 to x = 1 is given by:

$$s = \int_0^1 \sqrt{1 + \frac{1}{4}e^x} \; dx$$

6. Consider the sequence

$$b_n = n \sin\left(\frac{5}{n}\right),$$

defined for n $\geq$ 1. Does this sequence converge or diverge? In the case of convergence, find its limit.

7. Consider the series

$$\sum_{0}^{\infty} (-1)^n \frac{1}{1+\sqrt{n}}$$

Does this series converge absolutely, converge conditionally, or diverge?

Explain briefly.

8. Sum the series

$$\sum_{0}^{\infty} (-1)^n \frac{3}{2^n}$$

9. Find the first four *non-zero* terms of the Maclaurin series expansion of

$$x^4 \cos(x^2).$$

10. Evaluate the improper integral

$$\int_{0}^{\infty} \frac{32 \arctan x}{1+x^2} dx$$

.

11. Set up an integral for the area of the surface generated by revolving y = tan x, $0 \leq x \leq \pi/4$, about the x-axis. *Do not* evaluate. Sketch.

12. Express as a trigonometric integral. *Do not* evaluate.

$$\int x^{13}(4-x^2)^{\frac{9}{2}}\ dx$$

13. Consider the sequence

$$c_n = \left(\frac{e^n}{n!}\right)^2 + \frac{n^2 + 1}{(n+1)^2}$$

defined for n ≥ 1. Does this sequence converge or diverge? In the case of convergence, find its limit.

14. Convert the following parametric equation to an equation in Cartesian coordinates (i.e., express the curve in the form y = f(x)).

$x(t) = \tan t$, $y(t) = \sec^2 t - 1$

for $-\pi/2 < t < \pi/2$.

15. Evaluate $\int \frac{e^x}{e^{2x} + e^x - 12}\ dx$

16. Find the interval of convergence of the power series:

$$\sum_{1}^{\infty} \frac{1}{n(n+3)}\ (x-1)^n$$

You need not test the endpoints for convergence.

17. Solve the differential equation

$$\cosh x \, \frac{dy}{dx} + y \sinh x + e^x = 0$$

18. Evaluate

$$\int \sec\theta \tan\theta \sqrt{1+\sec\theta} \; d\theta.$$

19. Find the volume of the solid generated by revolving the region bounded by

$$y = \sqrt{9 - x^2} \;\; \textit{and} \;\; y = 0$$

about the x-axis. (You may use any method you wish.) Sketch.

20. Albertine, a mountain climber, is about to haul up a 50 m. length of hanging rope. How much work will she do if the rope weighs 0.625 N/m?

21. Evaluate

$$\int e^x \sec^2(e^x - 13) \; dx.$$

22. The region enclosed by the x-axis and the parabola $y = 3x - x^2$ is rotated about the vertical line x = -1. Using shells, write an integral that represents the volume of this solid. Sketch. *Do not* evaluate.

23. Does the following improper integral converge or diverge? Explain.

$$\int_{0+}^{\pi} \frac{dt}{\sqrt{t} + \sin t}.$$

24. Does the following improper integral converge or diverge? Explain.

$$\int_{1}^{\infty} \frac{x(x^2+1)^2(x^3+1)^3}{(x^4+1)^4}\,dx.$$

25. *Without using* l'Hôpital's rule, calculate:

$$\lim_{t\to 0} \frac{1-\cos t - \frac{t^2}{2}}{t^4}.$$

26. Does the following series converge or diverge? Explain.

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$

27. Does the following series converge or diverge? Explain.

$$\sum_{1}^{\infty} \frac{n+\cos^2 n}{n^3+n^2+n+1}$$

28. Consider the following recursively defined sequence:

$$a_1 = 1,\ a_2 = 4,\ and\ a_n = \frac{a_{n-1}+a_{n-2}+1}{3}\ \ for\ n \geq 3.$$

Find a_5.

29. Find the interval of convergence of the power series:

$$\sum_{1}^{\infty} \frac{1}{3^n+4^n}\ (x+11)^n$$

You need not test the endpoints for convergence.

30. The following probability density function

$$f(t) = \begin{cases} 0\ \ for\ t < 0 \\ \frac{1}{111} e^{-\frac{t}{111}}\ \ for\ t \geq 0 \end{cases}$$

models the length of time that a 60 watt bulb manufactured by Alphaville Electrical will last. Compute the probability that a random 60 watt bulb purchased from Alphaville Electrical will die within the first 25 hours of use. Express your answer to the nearest hundredth.

31. Let $f(x) = x^8 e^{2x}$. Compute $f^{(100)}(0)$. Do not simplify your answer.

32. Does the following series converge or diverge? Explain.

$$\sum_{n=1}^{\infty} \sqrt{e^{\frac{n}{n^2+1}}}$$

33. Find the sum of the following series:

$$\sum_{n=1}^{\infty} \left(\frac{1}{\ln(n+2)} - \frac{1}{\ln(n+1)} \right)$$

34. (a) Find the equation of the straight line in 3-space that passes through the point Q = (3, 4, 1) and is parallel to the vector 8 **i** – **j** + 9 **k**.

(b) Describe the set of points in 3-space that are 3 units from the z-axis.

(c) Describe the plane in 3-space that is perpendicular to the y-axis and passes through the point (9, 8, 0).

35. Evaluate $\int \frac{t^3+t+4}{t^2-9}\, dt.$

36. Solve the differential equation:

$$\frac{dy}{dx} = \frac{x - e^{-x}}{y + e^{y}}$$

Additional Extra Credit:

1. Find the length of one arch of the cycloid:

   x = a(θ – sin θ)

   y = a(1 – cos θ)

2. Evaluate the series without using a calculator!

$$\frac{(1)(2)(3)}{4} + \frac{(2)(3)(4)}{4^2} + \frac{(3)(4)(5)}{4^3} + \frac{(4)(5)(6)}{4^4} + \ldots$$

"Alice laughed: "There's no use trying," she said; "one can't believe impossible things."
"I daresay you haven't had much practice," said the Queen.
"When I was younger, I always did it for half an hour a day.
Why, sometimes I've believed as many as six impossible things before breakfast."

- Lewis Carroll, **Alice in Wonderland**