MATH 162 PRACTICE QUIZ III

1. Compute the value of each of the following convergent improper integrals:

(a) $\int_0^{\infty} e^{-5x}\ dx$

(b) $\int_0^{\infty} xe^{-x^2}\ dx$

(c) $\int_e^{\infty} \frac{1}{x(\ln x)^4}\ dx$

(d) $\int_{e^e}^{\infty} \frac{1}{x(\ln x)(\ln\ln x)^{1.1}}\ dx$

(e) $\int_{0+}^{1} \frac{1}{\sqrt{x}}\ dx$

(f) $\int_0^{1-} \frac{1}{\sqrt{1-x^2}}\ dx$

(g) $\int_0^{\infty} xe^{-x}\ dx$

2. For each of the following improper integrals, determine convergence or divergence. *Justify your answers!*

(a) $\int_e^{\infty} \frac{1}{\ln x}\ dx$

(*b*) $\int_e^{\infty} \frac{1}{x^{1/3}(\ln x)^4}\, dx$

(*c*) $\int_0^{\infty} \sqrt{x}\, e^{-x^2}\, dx$

3. For each of the following improper integrals, determine convergence or divergence. *Justify each answer! (That is, if you use the comparison test, exhibit the function that you choose to use for comparison and show why the appropriate inequality holds.)*

(a) $\int_0^{\infty} \frac{1+x+x^4}{(1+x)^5}\, dx$

(b) $\int_0^{\infty} \frac{1+x+e^x}{5+3e^{3x}}\, dx$

(*c*) $\int_e^{\infty} \frac{1+x+x^2+2012x^{99}}{1+(\ln x)^{91}+(x^5+1)^{21}}\, dx$

4. For which value(s) of the constant C will the following improper integral converge?

$$\int_4^{\infty} \left(\frac{2x}{x^2+1} - \frac{C}{2x+1} \right) dx$$

5. For each improper integral given below, determine convergence or divergence. (You may either perform the integration directly or else use the Comparison Test.) *Justify your answers!*

(a) $\int_0^{\infty} e^{-3x}\, dx$

(b) $\int_{19}^{\infty} \frac{x^3}{x^4+33}\, dx$

(c) $\int_{71}^{\infty} \frac{1}{\sqrt{x+13}}\, dx$

(d) $\int_3^{\infty} \frac{1}{(x+9)^{\frac{5}{4}}}\, dx$

(e) $\int_5^{\infty} \frac{1}{x(\ln x)\ln(\ln x)}\, dx$

(f) $$\int_5^{\infty} \frac{1}{x(\ln x)(\ln(\ln x))^{1.01}}\,dx$$

6. For each improper integral given below, determine convergence or divergence. (You will need to use the Comparison Test here.) *Justify your answers!*

(a) $$\int_0^{\infty} \frac{\sin^4 x}{(1+x)^2}\,dx$$

(b) $$\int_4^{\infty} \frac{1}{(\ln x)-1}\,dx$$

(c) $$\int_0^{\infty} \frac{(3+x)^2 + x\ln x + 5x + 1}{(1+9x+x^2)^4}\,dx$$

(d) $$\int_1^{\infty} \frac{\ln x}{x^3}\,dx$$

7. Find the volume of the solid of revolution obtained by rotating the curve $y = 1/(1 + x^2)^{1/2}$ from $x = 0$ to $x = \infty$ about the x-axis or explain why no such number exists.

8. For each of the following improper integrals, determine convergence or divergence. Use an appropriate version of the Comparison Test.

(a) $\displaystyle\int_{0+}^{\infty} \frac{1}{x^{\frac{2}{3}} + x^{\frac{4}{3}}}\, dx$

(b) $\displaystyle\int_{0+}^{\infty} \frac{1+x}{x^3 + \sqrt{x}}\, dx$

(c) $\displaystyle\int_{0}^{\frac{\pi}{2}-} \tan x \; dx$

(d) $\displaystyle\int_{0+}^{1} \frac{1-\ln x}{x^4}\, dx$

As far at the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality.

- Albert Einstein, **Sidelights on Relativity**