MATH 162 PRACTICE QUIZ 4A

1. For each of the following improper integrals, determine *convergence* or *divergence*. *Justify your answers!*

(a) $$\int_{0+}^{\infty} \frac{1+x}{x^3+\sqrt{x}}\,dx$$

(b) $$\int_{0}^{\frac{\pi}{2}-} \tan x\,dx$$

(c) $$\int_{0}^{2-} \frac{1}{\sqrt{4-x^2}}\,dx$$

(d) $$\int_{0+}^{1} \frac{1-\ln x}{x^4}\,dx$$

2. For each of the following sequences, determine *convergence* or *divergence*. In the case of convergence, find the *limit* of the sequence. *Briefly explain your reasoning!*

(a) $$c_n = \sqrt{1-\frac{3}{n}}$$

(b) $$d_n = \frac{\cos^4(n^3+n^2+5)}{n^2+23}$$

(c) $e_n = 1 + \arctan n$

(d) $u_n = \dfrac{e^n}{n^5}$

(e) $z_n = \dfrac{(n^5+1)^3(1-4n)^2}{(n+9)^{17}}$

3. Consider the following *recursively defined* sequence:

$a_1 = 4$

$a_2 = 2$

$a_n = a_{n-1}a_{n-2} - a_{n-1} - a_{n-2} + 1$ for $n \geq 3$.

Find the numerical values of a_3, a_4, a_5 and a_6. (Show your work.)

4. To which of the following series does the "*n^{th} term test for divergence*" apply? Explain!

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+5}$

(b) $\displaystyle\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^3}$

$$(e) \quad \sum_{n=1}^{\infty} \arctan(n)$$

$$(e) \quad \sum_{n=1}^{\infty} \arctan(n)$$

$$(f) \quad \sum_{n=1}^{\infty} n^{1/n}$$

Extra Credit: For n ≥ 1, let

$$a_n = \int_0^1 (x^2+2)^n \, dx.$$

Determine convergence or divergence of the sequence $\{a_n\}$. (*Hint:* Do *not* try to evaluate the integral! Calculator solutions are not accepted.)

There is more danger of numerical sequences continued indefinitely than of trees growing up to heaven. Each will some time reach its greatest height.

- Friedrich Ludwig Gottlob Frege, **Grundgesetz der Arithmetik** (1893)