MATH 162 PRACTICE QUIZ 4B

1. For each of the following improper integrals, determine *convergence* or *divergence*. *Justify your answers!*

(a) $$\int_{0+}^{1} \frac{-\ln x}{x} \, dx$$

(b) $$\int_{0+}^{\frac{\pi}{2}} \frac{\cos y}{\sqrt{\sin y}} \, dy$$

(c) $$\int_{0+}^{\infty} \frac{1}{x^{\frac{2}{3}} + x^{\frac{4}{3}}} \, dx$$

(d) $$\int_{0+}^{1} \ln t \, dt$$

2. For each of the following sequences determine *convergence* or *divergence*. In the case of convergence, determine the limit of the sequence as well. Briefly, justify each answer.

(a) $$a_n = \left(\frac{n}{1+n} \right)^n$$

(b) $$b_n = \frac{2^n + 3^n + 2008}{e^n + 1789}$$

(c) $c_n = \dfrac{123\sin(1+n) + 9\cos(3+n) + n^2 + 1234}{\sqrt{n^5 + 99}}$

(d) $d_n = \sqrt{1 + n + 14n^2 + 16n^4} - \sqrt{1 + 4n + n^2 + 16n^4}$

(e) $e_n = e^{-\frac{1+n}{3+5\ln n}}$

3. Consider the following *recursively defined* sequence:

$b_1 = 1$
$b_2 = 2$
$b_n = b_{n-1} + 5b_{n-2}$ for all $n \geq 3$.

Let $r_n = b_n/b_{n-1}$ and assume that the limit of r_n as $n \to \infty$ exists. Find this limit.

(Show your work.)

4. Which of the following infinite series are *geometric* and which are not? For those that are geometric, determine convergence or divergence. In the case of convergence, find the limit. Show your work!

$(a)\ \sum_{n=0}^{\infty} \dfrac{5^{n+1}}{9^{n-1}}$

$(b)\ \sum_{n=0}^{\infty} \dfrac{3^n 2^{2n+1}}{5^n}$

$(c)\ \sum_{n=0}^{\infty} \dfrac{2^n}{n^n}$

$(d)\ \sum_{n=0}^{\infty} \frac{13(2^{3n+7})}{9^n}$

$(e)\ \sum_{n=0}^{\infty} \frac{3^n+1}{4^n}$

Extra Credit: Determine whether the following improper integral converges or diverges. Show your work!

$$\int_0^{1^-} \sqrt{\frac{1+x}{1-x}}\ dx$$

Zeno-phobia: the irrational fear of converging sequences.

- Anonymous