MATH 162 PRACTICE QUIZ VII

1. Without using l'Hôpital's rule, find

$$\lim_{x \to 0} \frac{\ln(1+x) - x}{1 - \cos x}$$

2. By differentiating an appropriate power series, compute the following sum:

$$\sum_{n=1}^{\infty} \frac{n}{5^n}$$

3. By twice differentiating an appropriate power series, compute the following sum:

$$\sum_{n=1}^{\infty} \frac{n^2}{2^n}$$

4. Find the Taylor series of

$$F(x) = \cos \sqrt{x+1}$$

centered at x = -1.

Hint: Let u = x +1; find the Maclaurin series for cos $\sqrt{u}$.

5. Let F(x) = $x^4 \ln(1+x^2)$. Find $F^{(2072)}(0)$.

Hint: Beginning with a geometric series, find the Maclaurin series expansion of ln(1 + t).

6. State *Euler's identity*. Use Euler's identity to express cos (4x) in terms of cos x and sin x.

7. Solve the equation $z^4 = -1$.

8. Solve the equation $z^3 = i$.

9. Simplify each of the following, expressing each answer in the form a + bi.

(a) $3(2 – 5i) – 11(3 – 4i)$

(b) $(4 + 5i)(1 – 7i)$

(c) $\frac{1}{2 - i}$

(d) $\frac{2 + 3i}{1 - 2i}$

(e) $(1 + 2i)^2$

(f) $\overline{31 - 5i}$

10. Express each of the following in *polar form*: (a) $1 + i$, (b) $3 – 3i$, (c) $4 - 4\sqrt{3}\ i$, (d) $5 + 12i$

11. Solve the equation $z^5 = 1$. (You should have five solutions.)

12. Using Euler's formula, express sin 5x in terms of sin x and cos x. (*Hint:* $(a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$)

13. Without using L'Hôpital's rule, find

$$\lim_{x \to 0} \frac{e^{2x} - 4\cos\left(x^2\right) - 2x + 5 - 2e^{x^2}}{\sin(x^3) + x^5 e^x}$$

14. Show that $\cosh(x\,i) = \cos x$ and that $\sinh(x\,i) = i \sin x$.

15. Find the six *sixth roots* of 64.

16. Express the following integral as a power series:

$$\int_0^x t^2 e^{-t^2}\,dt$$

17. Find the first four terms of the binomial series of the function $(1 + x^2)^{-1/3}$.

18. Express as a numerical series:

$$\int_0^{0.2} \frac{e^{-x} - 1}{x}\,dx$$

19. Use an appropriate binomial series to find the first four non-zero terms of the Maclaurin series for arcsin x.

20. (Thomas) Verify the integration formula

$$\int e^{(a+bi)x}dx = \frac{a - bi}{a^2 + b^2}\ e^{(a+bi)x}\ +\ C,$$

where $C = C_1 + C_2 i$ is a complex constant of integration.

Using this formula, evaluate each of the following integrals

$$\int e^{ax}\cos bx\ dx\ and\ \int e^{ax}\sin bx\ dx$$

21. Express each of the following in the form a + bi

(a) i^{-1}

(b) $(-1)^i$

(c) $(1 + i)^{90}$

(d) $3e^{\pi i/6}$

(e) $\left(\sqrt{3} + i\right)^{11}$

(f) i^{i^i}

22. Using substitution (or any other method that you prefer), evaluate each of the following integrals:

$(a) \quad \displaystyle\int \frac{\tan(\ln x)}{x}\, dx$

$(b) \quad \displaystyle\int \sqrt{x}\, \sin(2x^{3/2})\, dx$

$(c) \quad \displaystyle\int e^x \sec^2(e^x - 13)\, dx$

$(d) \quad \displaystyle\int \sec x \tan x \sqrt{1 + \sec x}\, dx$

$(e) \quad \displaystyle\int \sqrt{a + b\sqrt{c + x}}\, dx$

$(f) \quad \displaystyle\int \frac{e^x \arcsin \sqrt{x}}{2\sqrt{x - x^2}}\, dx$

$(g) \quad \displaystyle\int \frac{dx}{\sqrt{\arctan x}\ (1 + x^2)}$