MATH 162 PRACTICE QUIZ VIII

1. Give the correct *form* (for example: A/(x+2) + B/(x –13)) for the partial fraction decomposition of the following rational functions. *Do not solve for A, B, C, …*

$$\frac{x^7 + x^3 - 9x + 2013}{(x^2 - 15x)\,(x^2 - 1)^2 (2x^2 + 5)^2}$$

2. Integrate each of the following functions:

   (a) $\tan^5 x$

   (b) $\sec^3 x \tan^5 x$

   (c) $\tan^6 2x$

   (d) $\sec^4 x$

   (e) $(\sec x)^{-4/3} \tan x$

   (f) $(\sin 10x)(\sin 5x)$

   (g) $\sin^9 x \cos^{11} x$

   (h) $(\cos 4x)(\sin 3x)$

3. Integrate each of the following functions:

   (a) $\dfrac{x^7 + 1}{x - 1}$

$(b)\quad \dfrac{x^2+2}{x^2-9}$

$(c)\quad \dfrac{x-3}{(x-1)(x-2)^2}$

$(d)\quad \dfrac{1}{(x^2+1)(x-2)^2}$

$(e)\quad \dfrac{e^x}{e^{2x}-5e^x+4}$

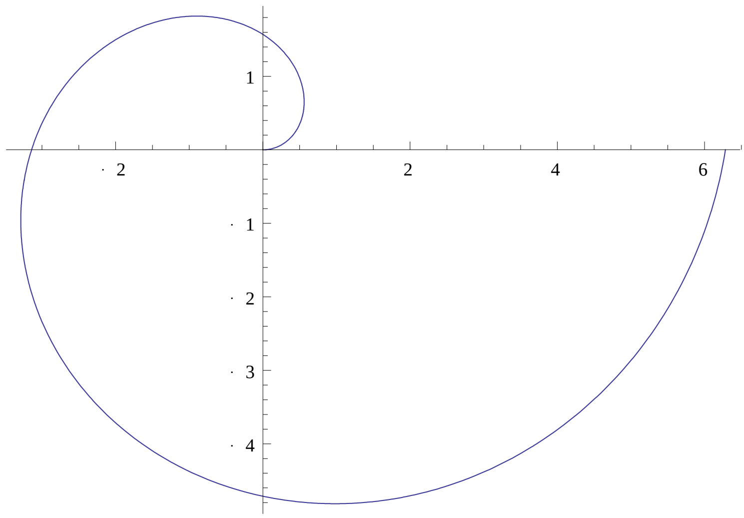
$(f)\quad \dfrac{\cos x}{2\sin^2 x+7\sin x+6}$

$(g)\quad \dfrac{\sinh x}{(\cosh x)^2-\cosh x}$

4. The following is the graph of the spiral of Archimedes,

$$r = \theta, \quad 0 \le \theta \le 2\pi.$$

Find the *area* of the region bounded by this curve and the positive x-axis.



5. Convert each polar equation to a Cartesian equation:

(a) $r^2 = 4r\cos\theta$

(b) $r = \dfrac{3}{4 - 5\cos\theta}$

6. Convert the given Cartesian equation to a polar equation:

(a) $x^2 - y^2 = 1$

(b) $x + 2y = 2013$

7. Find the arc length of the cardioid $r = 1 + \cos\theta$.

8. Find the area of the region bounded by the curve $r = e^{\theta}$ and the rays $\theta = \pi/6$, $\theta = \pi/3$.

9. Find the area of the region enclosed by the curves r = 1, r = 2 sin θ.

10. Show that each function given below is a solution to the corresponding differential equation:

$$(a)\quad y = -\frac{1}{x+3}, \qquad \frac{dy}{dx} = y^2$$

$$(b)\quad y = e^{-x}\arctan(2e^{x}), \qquad \frac{dy}{dx} + y = \frac{2}{1+4e^{2x}}$$

$$(c)\quad y = \frac{x}{\ln x}, \qquad x^2\frac{dy}{dx} = xy - y^2,\ x > 1$$

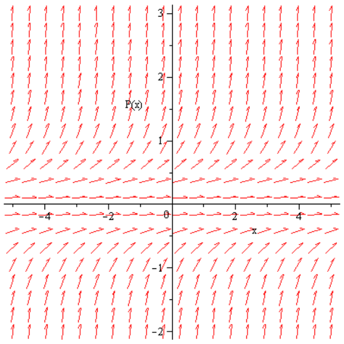
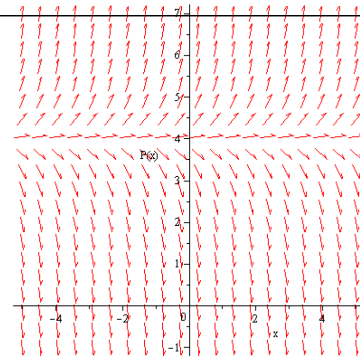
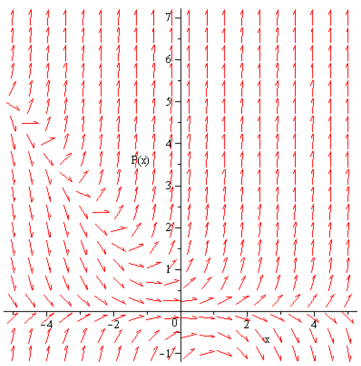
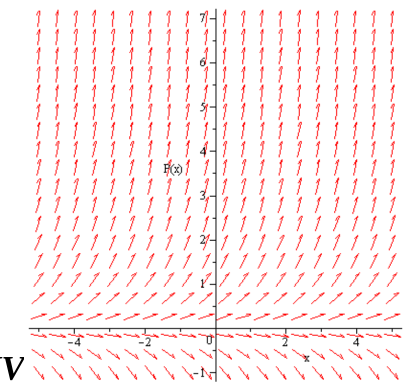
11. Match each of the following differential equations with its corresponding direction field.

(a) dy/dx = y

(b) dy/dx = 2(y – 4)

(c) dy/dx = y(x + y)

(d) dy/dx = y^2

 ***I***	 ***II***
 ***III***	***IV*** 

12. Using the change of variable v = y/x, solve the differential equation

$$\frac{dy}{dx} = \frac{y - 4x}{x - y}$$

13. Using the substitution v = y/x, solve the differential equation

$$2xy\frac{dy}{dx} = x^2 + y^2$$

14. Solve the following separable equations:

$$(a)\quad \frac{dy}{dx} = e^{3x-y}$$

$$(b)\quad \frac{dy}{dx} = 2x\sqrt{1-y^2}$$

$$(c)\quad \sec x \, \frac{dy}{dx} = e^{y+\sin x}$$

15. Solve the following initial value problems:

$$(a)\quad y^{-2}\frac{dx}{dy} = \frac{e^x}{e^x+1}, \quad y(0)=1$$

$$(b)\quad \frac{dy}{dx} = \frac{y\ln y}{x^2+1}, \quad y(0)=e^2$$

$$(c)\quad \frac{dy}{dx} = \frac{1}{e^{x+y+2}}, \quad y(0)=-2$$

16. Solve each of the following first-order linear equations:

$$(a)\quad x\frac{dy}{dx}+2y=1-\frac{1}{x},\quad x>0$$

$$(b)\quad e^{2x}\frac{dy}{dx}+2e^{2x}y=2x$$

$$(c)\quad x\frac{dy}{dx}-y=2x\ln x$$

17. Verify that $y_1(t) = 1$ and $y_2(t) = t^{1/2}$ are solutions of the second-order linear differential equation:

$$t^2\frac{d^2y}{dt^2}-2y=0\quad for\ \ t>0.$$

18. Verify that $y_1(x) = x$ and $y_2(x) = xe^x$ are solutions of the second-order linear differential equation:

$$x^2\frac{d^2y}{dx^2}-x(x+2)\frac{dy}{dx}+(x+2)y=0\quad for\ \ x>0.$$

19. For which value(s) of r is $y = e^{rx}$ a solution of the linear differential equation

$$4\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 3y = 0 \quad for \;\; x > 0.$$

I'm very good at integral and differential calculus,

I know the scientific names of beings animalculous;

In short, in matters vegetable, animal, and mineral,

I am the very model of a modern Major-General.

About binomial theorems I'm teeming with a lot of news,

With many cheerful facts about the square on the hypotenuse.

- W. S. Gilbert, **The Pirates of Penzance**(1879)