MATH 162 PRACTICE TEST 1A

1. Use integration by parts to evaluate the following integral:

$$\int (\ln x)^2 \, dx$$

2. The base of a solid is the region enclosed by y = 1/x, y = 0, x = 1, and x = 4. Every cross section of the solid taken perpendicular to the x-axis is an isosceles right triangle with its hypotenuse across the base. Express the volume of the solid as a Riemann integral. (You need not evaluate your integral.)

3. A cylindrical barrel, standing upright on its circular end, contains muddy water. The top of the barrel, which has a diameter of 3 feet, is open. The height of the barrel is 4 feet and it is ¾ filled with muddy water. The weight of the muddy water at a distance of *h* feet from the bottom of the barrel is given by w(h) = 51 + k(4-h) pounds/feet^3, where *k* is a positive constant. Find the total work done in pumping the muddy water to the top rim of the barrel. (*Note:* Evaluate your integral. Your answer will include the constant *k*.)

4. A snail crawls along the curve $y = x^{3/2}$ at a speed of 3 feet per hour. How long does it take the snail to travel from the point (1, 1) to the point (4, 8)? Give a numerical answer.

5. Using implicit differentiation, find a formula for the derivative of each of the following inverse functions:

(A) arcsinh x

(B) arccosh x

(C) arctanh x

6. A cable that weighs 2 lb/ft is used to lift 800 lb of coal up a mineshaft 500 ft deep. Find the work done. Express your answer numerically.

7. Consider the region, R, bounded by the curves $y = \ln x$, $y = 0$, and $x = 2$. Suppose that R is rotated about the line $x = -1$. Express the volume of this solid of revolution as a Riemann integral using each of the following methods. Be certain to make a sketch for each.

(A) cylindrical shells

(B) washers

8. The region in the first quadrant bounded by the curves

$$y = (1 + x)^{1/2},\ y = 0,\ x = 0, \text{ and } x = 1$$

is rotated about the x-axis. Find the surface area of this solid of revolution. Express your answer numerically.

9. True or False? Justify each answer.

(A) $x(3x + 1)^9 = O(x^{10})$

(B) $1/x = o(1/x^2)$

(C) $1/x^4 = o(1/x^3)$

(D) $(1 + \ln x)^{21} = o(x)$

(E) $e^x = o(x^x)$

10. Suppose that the volume of water required to fill a hollow object to a depth of *h* inches (where $0 \le h \le \pi/2$) is given by the function:

$$V(h) = 1.5h + \sin h \quad \text{cubic inches.}$$

What is the cross-sectional area of the object 1 inch above its base?