MATH 162 PRACTICE TEST 1B

1. Find the area of the surface obtained by rotating the triangle with vertices $(0, 0)$, $(2, 0)$, $(1, 3^{1/2})$ about the axis $y = -3$.

2. Using integration by parts, evaluate

$$\int x \sec^2 x \, dx.$$

3. Suppose that an inverted conical tank is filled with kerosene (weighing 51.2 lb/ft^3) and that the tank measures 30 ft high and has radius at the top of 10 ft. Compute how much work would be done in pumping all of the kerosene over the top of the tank. (You may express your answer as a Riemann integral.)

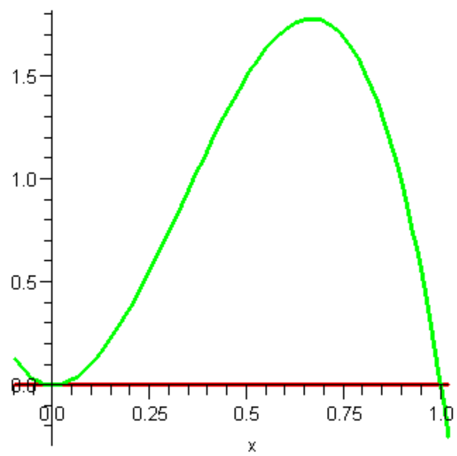
4. Express the arc length of the ellipse $\frac{x^2}{16} + \frac{y^2}{49} = 1$ as a Riemann integral. (*Hint:* First parameterize the ellipse using $x = 4 \cos t$ and $y = 7 \sin t$ and then select an appropriate interval of t values.)

5. The base of a solid is the region between the curve $y = (\sin x)^{1/2}$ and the interval $[0, \pi]$ on the x-axis. The cross sections perpendicular to the x-axis are equilateral triangles with bases running from the x-axis to the curve $y = (\sin x)^{1/2}$. Write a Riemann integral that expresses the volume of the solid. (You need not evaluate the integral.)

6. Consider the region in the xy-plane bounded by the curves $y = 4 - x^2$ and $y = 2 - x$. If this region is rotated about the x-axis, express the volume of this solid of revolution as a Riemann integral. (You need not evaluate the integral.)

7. Consider the region in the xy-plane bounded by the curves $y = x^4$ and $y = 2$. If this region is rotated about the line $y = 13$, express the volume of this solid of revolution as a Riemann integral. (You need not evaluate the integral.)

8. Consider the region in the xy-plane bounded by the curves $y = 12(x^2 - x^3)$ and the x-axis as illustrated below. If this region is rotated about the line $x = 2$, express the volume of this solid of revolution as a Riemann integral. You need not evaluate the integral. (Hint: Use the method of cylindrical shells.)



9. Let S be the surface of revolution obtained by rotating the curve

$y = e^{-2x^2}, \quad 0 \le x \le 3,$ about the x-axis. Find a Riemann integral that expresses the surface area of this region. (You need not evaluate the integral.)

10. A 35 foot chain with mass density 4 lb/ft is initially coiled on the ground. How much work is performed in lifting the chain vertically so that it is fully extended (with one end touching the ground)? You may express your answer as a Riemann integral.

11. Using the definitions of hyperbolic functions, verify that

$$\cosh 2x = (\cosh x)^2 + (\sinh x)^2.$$

12. Using *integration by parts*, find an anti-derivative of each if the following functions:

(a) xe^x

(b) $\sin(\ln x)$

(c) $x \sinh(3x)$

(d) arc sin x

13. *True or False*. Justify your answers:

(a) $x = o(x + \ln x)$

(b) $\arctan x = O(1)$

(c) $\ln x = o(x+1)$

(d) $1/x^2 + 1/x^4 = O(1/x^2)$

(e) $\cosh x = O(e^x)$

(f) $1/x^2 + 1/x^4 = O(1/x^4)$

There is something I don't know
that I am supposed to know.
I don't know what it is I don't know,
and yet am supposed to know,
and I feel I look stupid
if I seem both not to know it
and not know what it is I don't know.
Therefore I pretend I know it.

- R. D. Laing, **Knots**