MATH 162 PRACTICE TEST 2-A

1. Determine *convergence* or *divergence* of each of the following improper integrals:

(A) $$\int_{0+}^{\frac{1}{e}} \frac{(-\ln x)}{x^4}\,dx$$

(B) $$\int_{0+}^{\infty} \frac{1}{\sqrt{x+x^4}}\,dx$$

2. For each of the following infinite series, determine *convergence* or *divergence*. In the case of convergence, find the sum of the series:

(a) $$\sum_{n=1}^{\infty} \ln\frac{n+1}{n}$$

(b) $$\sum_{n=0}^{\infty} \frac{5}{9^n}$$

(c) $$\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^n$$

(d) $$\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$$

(e) $\sum_{n=1}^{\infty} \cos\left(\frac{5}{n}\right)$

(f) 0.123123123…

3. *Evaluate* each of the following convergent improper integrals. Show your work!

(A) $\int_0^{\infty} t^3 e^{-t^4}\, dt$

(B) $\int_3^{\infty} \frac{1}{x(1+\ln x)^{7/3}}\, dt$

4. For each of the following improper integrals, determine convergence or divergence. *Justify each answer! (That is, if you use the comparison test, exhibit the function that you choose to use for comparison and show why the appropriate inequality holds.)*

(A) $\int_0^{\infty} \frac{1+x+x^4}{(1+x)^5}\, dx$

(B) $\int_0^{\infty} \frac{1+x+e^x}{5+3e^{3x}}\, dx$

5. Select any five of the following six sequences. For each sequence chosen, determine *convergence* or *divergence*. Justify your answers. Calculator results will not earn full credit. You may answer all six to earn extra credit.

(a) $a_n = \dfrac{100^n + 23^n}{n! + 7^n}$

(b) $b_n = \left(1+\dfrac{4}{n}\right)^n$

(c) $c_n = \dfrac{\ln(n+2013\pi)}{\ln(n)}$

(d) $d_n = \dfrac{\cos\left(\dfrac{\pi}{n}\right)}{n}$

(e) $e_n = \displaystyle\int_0^n e^{-\pi t}\, dt$

(f) $f_n = \sqrt{\dfrac{n+1}{n} + \dfrac{\sin(n^2)}{n^2} + e^{-\frac{3}{n}} + \dfrac{e^n}{\sinh n}}$

6. Find the sum of each of the following convergent series. Show your work.

(a) $\displaystyle\sum_{n=0}^{\infty} \left(e^{-n} - e^{-n-1}\right)$

(b) $\displaystyle\sum_{k=0}^{\infty} \frac{(-4)^{k+1}}{5^{k-1}}$

(c) $0.314314314314314\ldots$

7. (a) Give an example of a numerical series that is *not positive* but which is *absolutely convergent.*

(b) Give an example of a *conditionally convergent* numerical series.

(c) Give an example of two *divergent* numerical series whose sum is *convergent.*

8. For each series below, determine convergence or divergence. Justify each answer.

(a) $\displaystyle\sum_{m=1}^{\infty} \left(\frac{e}{m}\right)^{m}$

(b) $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{1}{5 + n^{7} \ln n}\right)$

(c) $\sum_{n=3}^{\infty} (-1)^n \frac{5}{\ln n}$

(d) $\sum_{n=1}^{\infty} \frac{(n!)^2 3^n}{(2n+1)!}$

(e) $\sum_{k=1}^{\infty} (-1)^k \frac{k(k+1)(k^2+5)}{(k-13\ln k)^4}$

(f) $\sum_{n=1}^{\infty} \frac{2^n}{\left(1+\frac{1}{n}\right)^{n^2}}$

9. Consider the following recursively defined sequence:

$c_1 = 7$, $c_2 = 4$, and

$$c_{n+1} = \frac{\left(c_n + \frac{3}{(c_{n-1})^2}\right)}{2} \quad for \;\; n \geq 2$$

(a) Find the values of c_3, c_4 and c_5.

(b) Assuming that the limit of c_n (as n $\to \infty$) exists, find its value.

10. *Find* $\lim_{n\to\infty} n^{\frac{1}{\ln n}}$ (Show your work!)

"Can you do addition?" the White Queen asked. "What's one and one and one and one and one and one and one and one and one and one?" "I don't know," said Alice. "I lost count."

- Lewis Carroll, **Through the Looking Glass**