MATH 162 PRACTICE TEST 2-B

1. For each series below, determine convergence or divergence. Justify each answer.

(a) $\displaystyle\sum_{m=1}^{\infty} \frac{1}{\sqrt{m+5}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{10^{n}(n!)^{3}}{(3n)!}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n}}{n}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{3-\cos^{3} n}{n(2+\cos n)}$

2. *For each sequence below, determine convergence or divergence. In the case of convergence, find the limit. Justify your answer.*

1. $\displaystyle\frac{11^{n}}{9^{2n-3}}$

2. $\left(\frac{n}{n+1}\right)^n$

3. $n^2 \sin(\frac{3}{n^2})$

4. $\frac{\sqrt{n^4+n^3+n+13}}{(3n+11)^2}$

5. $\sqrt{n^2+9n+7}-\sqrt{n^2+5n-13}$

6. $\frac{(\ln n)^8}{n}$

3. *Consider the following recursively defined sequence:*

$b_1 = 7$,

$$b_{n+1} = \frac{\left(b_n + \frac{7}{b_n}\right)}{2} \quad \text{for } n \geq 1$$

(a) Find the values of b_2 and b_3.

(b) Assuming that the limit of b_n as $n \to \infty$ exists, find its value.

4. *For each of the following improper integrals, determine convergence or divergence. Justify each answer!*

(A) $$\int_{0+}^{\frac{1}{3}} \frac{1}{x(\ln x)^2}\, dx$$

(B) $$\int_{0+}^{\infty} \frac{1}{\sqrt[3]{x+x^6}}\, dx$$

(C) $$\int_{0+}^{\infty} \frac{1+x+x^2}{x^4+x}\, dx$$

(D) $$\int_{0}^{\frac{\pi}{2}-} \sec^2 x\, dx$$

5. *For each of the following sequences, determine convergence or divergence. In the case of convergence, find the limit of the sequence.*

(A) $$x_n = e^{\frac{1}{n}}$$

(B) $$y_n = \frac{n!}{n+1}$$

(C) $$z_n = \frac{\sin n}{n} + \frac{5}{n}$$

(D) $$c_n = \frac{3(2n+1)^3}{(1-n)^2(4n+13)}$$

(E) $$a_n = \sec\left(\ln\left(\sin^4\left(\frac{\pi}{2} + \frac{1}{n^2}\right)\right)\right)$$

6. For each of the infinite series below, determine *convergence* or *divergence*. In the case of convergence, compute the sum of the series. Be certain to justify your answers!

(a) $$\sum_{n=0}^{\infty} \frac{13^n}{e^n}$$

(b) $$\sum_{n=1}^{\infty} \sec\left(e^{-n^2}\right)$$

(c) $$\sum_{n=1}^{\infty} \frac{3^{2n+1}}{5^{n-1}}$$

(d) $$\sum_{n=1}^{\infty} \frac{\left(1+n^2\right)^2}{n^3+99n^2+101\sqrt{n}+13}$$

(e) $$\sum_{n=1}^{\infty} \cos\left(\frac{2013}{n}\right)$$

(f) $$\sum_{n=1}^{\infty} \frac{n^4+n^2+13}{(n^2+4)^2}$$

(g) 0.04040404…

7. For each improper integral given below, determine convergence or divergence. (No need to use the Comparison Test here.) *Justify your answers!*

(A) $$\int_0^{\infty} e^{-2012x}\, dx$$

(B) $$\int_{19}^{\infty} \frac{x^3}{x^4+33}\, dx$$

(C) $$\int_{71}^{\infty} \frac{1}{\sqrt{x+13}}\, dx$$

(D) $$\int_3^\infty \frac{1}{(x+9)^{13/11}}\,dx$$

(E) $$\int_5^\infty \frac{1}{x(\ln x)\ln(\ln x)}\,dx$$

(F) $$\int_5^\infty \frac{1}{x(\ln x)(\ln(\ln x))^{1.01}}\,dx$$

8. For each improper integral given below, determine convergence or divergence. (You will need to use the Comparison Test here.) *Justify your answers!*

(A) $$\int_0^\infty \frac{\sin^{2013}(3+5x)}{(2012+x)^2}\,dx$$

(B) $$\int_4^\infty \frac{1}{(\ln x)-1}\,dx$$

(C) $$\int_0^\infty \frac{(3+x)^2+133x\ln x+5x+1}{(1+99x+x^2)^4}\,dx$$

(D) $$\int_1^{\infty} \frac{\ln x}{x^3}\, dx$$

9. Find the volume of the solid of revolution obtained by rotating the curve $y = 1/(1 + x^2)^{1/2}$ from $x = 0$ to $x = \infty$ about the x-axis or explain why no such number exists.

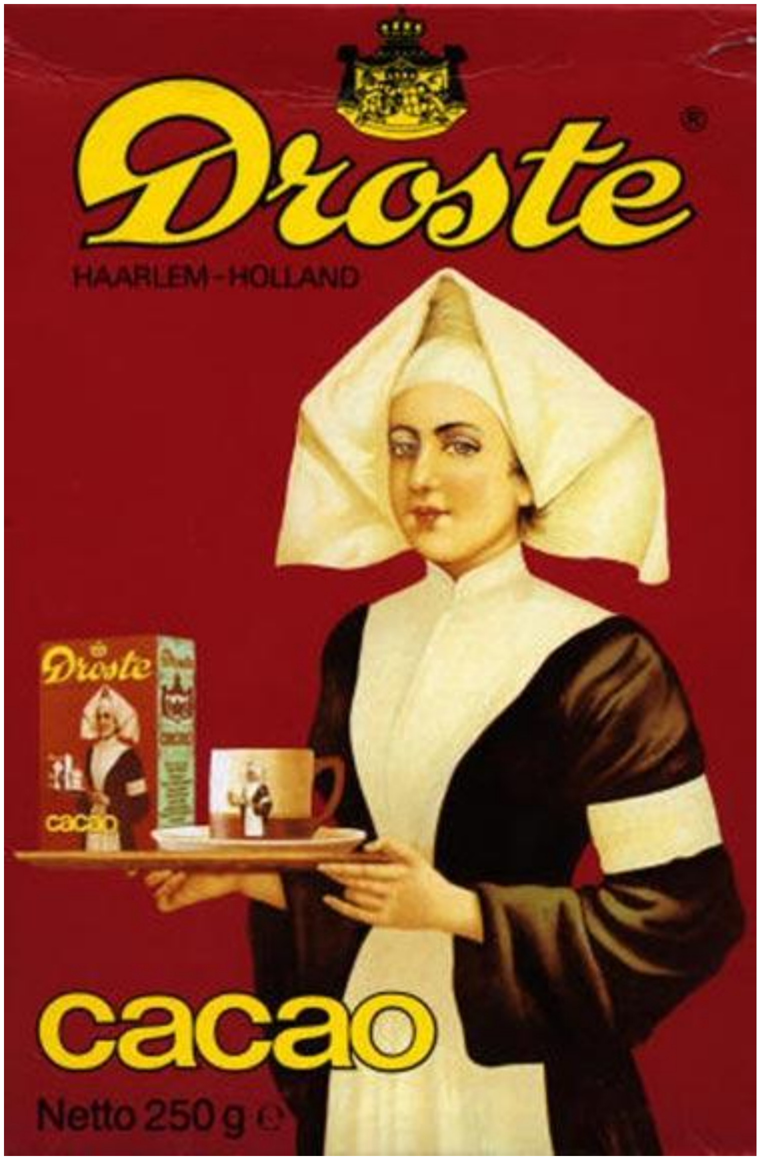
Extra extra credit:

For which values of p and q does the following improper integral converge?

$$\int_{0+}^{\infty} \frac{1}{x^p + x^q}\, dx$$

My New Zoo, McGrew Zoo, will make people talk.
My New Zoo, McGrew Zoo, will make people gawk
At the strangest odd creatures that ever did walk.
I'll get, for my zoo, a new sort-of-a hen
Who roosts in another hen's topknot, and then
Another one roosts in the topknot of his,
And another in his, and another in HIS,
And so forth and upward and onward, gee whiz?

- Dr. Seuss, **If I Ran the Zoo**


Droste
HAARLEM - HOLLAND
cacao
Netto 250 g ℮